

UNCLASSIFIED

AD NUMBER

ADB011931

LIMITATION CHANGES

TO:

Approved for public release; distribution is unlimited.

FROM:

Distribution: Further dissemination only as directed by Dept. of Defense, Attn: Public Affairs Office, Washington, DC 20301, 03 MAR 1965, or higher DoD authority.

AUTHORITY

DoD ltr 12 Nov 1976

THIS PAGE IS UNCLASSIFIED

THIS REPORT HAS BEEN DELIMITED
AND CLEARED FOR PUBLIC RELEASE
UNDER DOD DIRECTIVE 5200.20 AND
NO RESTRICTIONS ARE IMPOSED UPON
ITS USE AND DISCLOSURE.

DISTRIBUTION STATEMENT A

APPROVED FOR PUBLIC RELEASE;
DISTRIBUTION UNLIMITED.

THE **BOEING** COMPANY

CODE IDENT. NO. 81205

DOCUMENT NO.

14
D2-36251-1-REV-A

TITLE

ANALYSIS OF THE CHEMICALLY-REACTIVE MIXING REGION

BETWEEN A TURBULENT ROCKET EXHAUST AND A CONFINED AIR STREAM

MODEL

Research

CONTRACT NO.

ISSUE NO.

ISSUED TO

10 Donald L. Emmons

12 110p.

11 3 Mar 65

SEE DISTRIBUTION LIMITATIONS PAGE

PREPARED BY

D. L. Emmons

2/5/65

SUPERVISED BY

J. A. Brousseau

3/8/65

APPROVED BY

J. A. Brousseau

3/8/65

APPROVED BY

APPROVED BY

DDC

JUN 30 1976

D

BOEING

NO. D2-36251-1

SECT

PAGE 1

REV. SYM

059600

2-5145

U3 4287-9000 REV. 9/63

AD No.

DDC FILE COPY

ADB011931

3-16-65-

☐ UNLIMITED - TO ALL AGENCIES OF THE DEPARTMENT OF DEFENSE AND THEIR CONTRACTORS.

THIS REPORT MAY BE DISTRIBUTED TO NONMILITARY AGENCIES NOT APPROVED ABOVE SUBJECT TO BOEING APPROVAL OF EACH REQUEST.

NOTE: THE LIMITED CATEGORY MAY BE CHECKED ONLY BECAUSE OF ACTUAL OR POTENTIAL
PATENT, PROPRIETARY, ETHICAL, OR SIMILAR IMPLICATIONS.

- #1 ☒ Distribution of this report is Boeing-controlled. Qualified DDC users shall request through The Boeing Company, Aero-Space Division, Box 3707, Seattle, Washington 98124, _____.
- #2 ☐ U. S. Government agencies may obtain copies of this report directly from DDC. Other qualified DDC users shall request through The Boeing Company, Aero-Space Division, Box 3707, Seattle, Washington 98124, Attention:_____.
- #3 ☐ U. S. Military agencies may obtain copies of this report directly from DDC. Other qualified users shall request through The Boeing Company, Aero-Space Division, Box 3707, Seattle, Washington 98124, Attention:_____.
- #4 ☐ Qualified requesters may obtain copies of this report from DDC.

[illegible]**BOEING**

NO. D2-36251-1

SECT.

PAGE 2

DDC
RECEIVED
JUN 30 1976
RECEIVED
D

ACTIVE PAGE RECORD

SECTION	ORIG REL PAGE NO.	REV SYM	ADDED PAGES					SECTION	ORIG REL PAGE NO.	REV SYM	ADDED PAGES				
			PAGE NO.	REV SYM	PAGE NO.	REV SYM	PAGE NO.				PAGE NO.	REV SYM	PAGE NO.	REV SYM	PAGE NO.
	1								49						
	2								50						
	3 ^{3.1}	A							51						
	4	A							52						
	5								53						
	6								54						
	7								55						
	8								56						
	9								57						
	10								58						
	11								59						
	12								60						
	13 ^{3.1}								61						
	14	A							62						
	15								63						
	16								64						
	17								65						
	18								66						
	19	A							67						
	20	A							68						
	21								69						
	22								70						
	23								71						
	24								72						
	25	A							73						
	26								74						
	27								75						
	28	A							76						
	29								77						
	30	A							78						
	31								79						
	32								80						
	33								81						
	34	A							82						
	35	A							83						
	36								84						
	37								85						
	38								86						
	39								87						
	40								88						
	41								89	A					
	42								90						
	43								91						
	44								92						
	45								93						
	46														
	47														
	48														

U3 4801 0600 ORIG. 8/62

2-5142-2

REV SYM A

BOEING

NO. D2-36251-1

SECT.

PAGE 3

ACTIVE SHEET RECORD												
ORIG. REL SHT NO.	REV LTR	ADDED SHEETS					ORIG. REL SHT NO.	REV LTR	ADDED SHEETS			
		SHT NO.	REV LTR	SHT NO.	REV LTR				SHT NO.	REV LTR	SHT NO.	REV LTR
94												
95												
96												
97												
98												
99												
100												
101												
102												
103												
104												
105												
106												
107												
108												
109												

REV LTR _____

BOEING	NO.	D2-36251-1
	SH.	3.1

REVISIONS			
SYM	DESCRIPTION	DATE	APPROVED
A	The following pages were revised: 3, 4, 15, 19, 20, 25, 28, 30, 34, 35, 89	10/14/68 11/26/68	JRO

U3 4287 9025 ORIG. 8/62

2-5142-2

REV SYM A

BOEING

NO. D2-36251-1

SECT.

PAGE 4

ABSTRACT & KEY WORD

A theoretical analysis was developed for predicting the flow properties in the mixing region between a particle-laden, turbulent rocket exhaust and a surrounding air stream. It was assumed that the turbulent boundary layer equations, modified to account for particles, were valid within the mixing region. In treating the chemical aspects of the problem it was assumed that the flow was in equilibrium in accordance with three chemical reaction equations. The chemical species comprising the flow were limited to the following: H_2O , H_2 , O_2 , CO , CO_2 , N_2 , HCl , Al , Al_2O_3 , and one additional inert species which remained as an input selection. The phenomenological model employed for the turbulent transport coefficients is discussed in detail and compared with various other models.

The solution of the partial differential equations was obtained by transforming the equations using the von Mises transformation, expressing the transformed equations in finite-difference form, and solving the resulting equations utilizing a computer program developed for the SRU 1107.

Results from the computer program were successfully compared with experimental results obtained from air-augmentation, free jet, and fuel injection experiments. Output data from the computer program comprises velocity, temperature, density, species concentration, and Mach number profiles at various axial locations along the mixing region.

Gas-Particle Flow
Mixing Analysis
Chemical Reactions

Turbulent Flow
Eddy Transport Coefficients
Air Augmented Rocket
Supersonic Combustion

TABLE OF CONTENTS

	<u>PAGE</u>
LIST OF ILLUSTRATIONS	8
NOTATION	9
1.0 INTRODUCTION	11
1.1 General Remarks	11
1.2 Scope of Present Analysis	12
1.3 Computer Program	14
2.0 DERIVATION OF THE GOVERNING EQUATIONS	16
2.1 Fundamental Equations	16
2.1.1 General	16
2.1.2 Global Continuity Equation	18
2.1.3 Species Continuity Equations	21
2.1.4 Momentum Equations	22
2.1.5 Energy Equation	26
2.1.6 Equation of State	27
2.1.7 Alternate Forms of the Fundamental Equations	28
2.2 Transformation of the Fundamental Equations	30
2.3 Development of the Finite Difference Equations	35
2.4 Initial and Boundary Conditions	41
3.0 TURBULENT TRANSPORT COEFFICIENTS	41
3.1 General	41
3.2 Incompressible Theory	45
3.3 Compressible Theory	49
3.4 Turbulent Laws and Prandtl Numbers	52
4.0 CHEMICAL MODEL	52
4.1 General	52
4.2 Nonequilibrium Chemistry	55
4.3 Equilibrium Chemistry	58
4.4 Comparison of Equilibrium and Nonequilibrium Chemistry	60
5.0 COMPUTER PROGRAM	60
5.1 Capability	60
5.2 Pressure or Duct Profile Option	60

TABLE OF CONTENTS

	<u>PAGE</u>
5.3 Reference Streamline	61
5.4 Mixing Zone Definition	63
5.5 Stability Criteria	64
5.6 Determination of Initial Grid Size	66
5.7 Input Procedure	67
6.0 REFERENCES	74
7.0 APPENDICES	78
7.1 Computer Program Notation	78
7.2 Comparison of Theory and Experiment	81
7.3 Sample Problems	87
DOCUMENT CONTROL DATA - R&D	108

LIST OF ILLUSTRATIONS

<u>FIGURE</u>		<u>PAGE</u>
1.	Two-Dimensional Mixing Flow Model	13
2.	Control Volume for Gas-Particle Mixtures	17
3.	Typical Flow Field	31
4.	Finite-Difference Grid Network	32
5.	Flow Regimes for Confined Mixing	37
6.	Experimental and Theoretical σ -Values for Free Jets	46
7.	Theoretical Effect of Velocity Ratio on Similarity Parameter Ratio	48
8.	Effect of Reaction Mechanism and Rate Constants on Theoretical Static Temperature Distribution	59
9.	Comparison of Theoretical Temperature Profiles for Non- Equilibrium Flow	59
10.	Location of Reference Streamline Coincident with Plume Slipline	62
11.	Input Form for Computer Program	68
12.	Comparison of Theory and Experiment for Free Jet Velocity Profiles	82
13.	Comparison of Theory and Experiment	84
14.	Comparison of Theory and Experiment	86
15.	Input Form for Sample Problem	93
16.	Sample Output Data	94

NOTATION

A	area, ft^2
b	width of mixing zone, ft.
c	constant in eddy viscosity expression, ft.
C_i	concentration of species i, lbm of i/lbm of mixture
C_p	specific heat at constant pressure, B/lbm-°R
D	molecular diffusion coefficient, ft^2/sec
e	internal energy, B/lbm
g	gravitational constant, $32.17 \text{ lbm-ft/lbf-sec}^2$
h	static enthalpy, B/lbm
Δh_f°	heat of formation, B/lbm
J	mechanical equivalent of heat, 778 ft-lb/B
K_p	equilibrium constant
k_f	forward reaction rate constant, $\text{ft}^3/\text{lb-mole sec}$
L	reference length, ft.
l	Prandtl's mixing length parameter, ft.
Le	turbulent Lewis number
\dot{m}	mass flux, $\text{lbm}/\text{ft}^2\text{-sec}$
m_p	particle density, lbm of p/ ft^3 of
M	Mach number
n	exponent in eddy viscosity expression
P	static pressure, psfa
Pr	turbulent Prandtl number
R_u	universal gas constant, 1544 ft-lb/lb-mole °R

r	radius, ft.
Sc	turbulent Schmidt number
T	static temperature, °R
u	axial velocity component, ft/sec
v	radial velocity component, ft/sec
\dot{W}_i	net rate of production of species i , lbm/ft ³ -sec
W_i	molecular weight of species i , lbm/lb-mole
x	axial distance, ft.
y	radial distance, ft.
δ	exponent for two-dimensional ($\delta = 0$) or axisymmetric flow ($\delta = 1$)
ϵ	eddy diffusivity, ft ² /sec
ρ_g	gas density, lbm of gas/ft ³ of gas
ρ_p	particle density, lbm of p /ft ³ of gas
σ	similarity parameter
τ	shear stress, lb/ft ²
ψ	stream function, lbm/sec

Subscripts

D	mass diffusivity
g	gas
H	heat diffusivity
i	species
m, n	grid point coordinates
p	particle
V	momentum diffusivity

1.0 INTRODUCTION

1.1 General Remarks

Analytical studies involving the turbulent mixing of multi-component gases are required in a number of flow problems. Examples of such problems are those pertaining to base heating, wakes behind bluff bodies, loss of communication due to plume attenuation, and most recently, air augmentation. Fundamental studies of the mixing process between two moving streams have been conducted over the past several years with varying degrees of success. However, it has been only recently, with the advent of high-speed computers, that significant progress has been achieved in analytically predicting the properties within turbulent mixing regions. An excellent monograph describing the recent theoretical developments pertaining to turbulent jet mixing has been compiled by Abramovich (1)*.

Recent air-augmentation studies indicate that accurate prediction of engine performance requires a detailed study of the mixing region between a rocket exhaust jet and a confined secondary air stream. Since rocket exhaust gases, in general contain excess fuel, not only must the mixing process be analyzed, but also chemical reactions must be taken into consideration. The problem is further complicated by the fact that the entire flow is confined within a duct, thus necessitating the consideration of axial pressure gradients. Analyses of flows similar to those found in this type of problem have recently been developed by Libby (2) and Vasiliu (3). Libby, for example, has analyzed the unconfined mixed flow field for supersonic combustion studies assuming equilibrium chemistry; whereas

*Numbers in parentheses refer to references listed in the Bibliography

Vasiliu assumed nonequilibrium chemistry in his analysis of the mixing between a rocket exhaust jet and a supersonic air stream. Mikhail (4), on the other hand, has treated the mixing of coaxial incompressible streams in a duct but did not consider chemical reactions.

1.2 Scope of Present Analysis

The analysis and associated computer program described herein were developed specifically for the air-augmentation problem. However, it should be noted that the computer program may be employed for other problems. The primary objective of the analysis was to provide a method for calculating the velocity, temperature, and composition profiles in the mixing region between a fuel-rich rocket exhaust and an air stream confined within a duct of specified geometry. Since the exhaust of solid propellant rockets often contains solid and/or liquid particles, the analysis was developed for either gas-particle systems, or systems consisting solely of gaseous species. In treating the chemical aspects of the problem it is assumed that the flow is in chemical equilibrium in accordance with three chemical reaction equations; or, if desired, the flow may be treated as frozen. The total number of chemical components comprising the flow is limited to a maximum of ten species.

The flow model employed for the analysis is presented in Figure 1. At the periphery of the rocket nozzle exit, where the air stream and rocket exhaust first come into contact, a mixing region is assumed to begin. As the flow moves downstream, the mixing region widens on both sides of the plume slipline - the inner boundary approaching the duct centerline and the outer boundary the duct wall. At some distance downstream from the nozzle

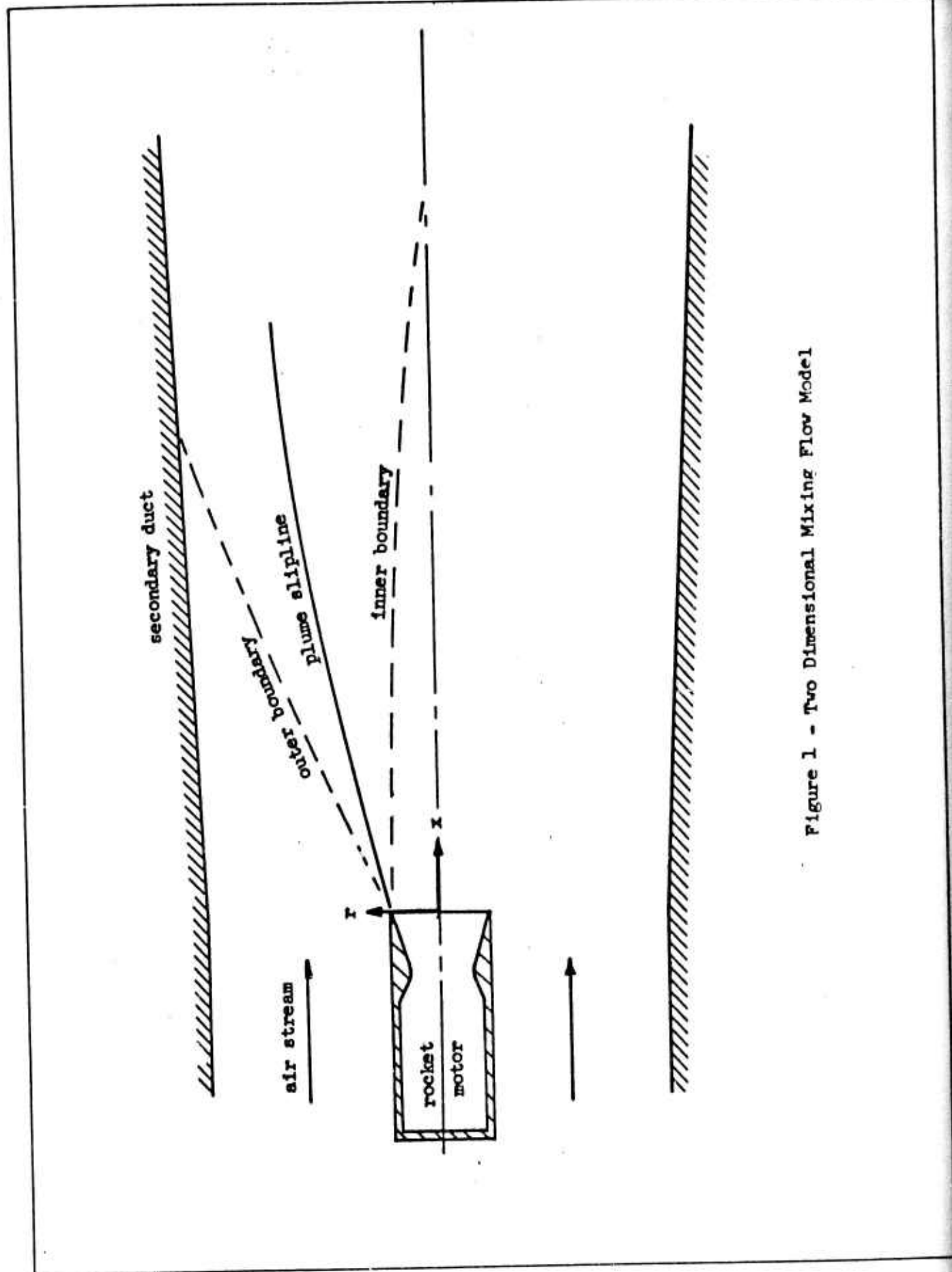


Figure 1 - Two Dimensional Mixing Flow Model

REV LTR

U3 4286-2000 REV. 6/64

BOEING

NO.

D2-36251-1

SH.

13

the mixing region may be considered to extend across the entire duct. In establishing the coordinates of the plume slipline the method of characteristics solution is employed (5). In the areas outside the mixing region, the flow is considered to be inviscid with uniform properties in the radial direction. Within the mixing region turbulent transport processes are assumed to prevail with all molecular transfer considered negligible. This assumption is justified for the problem under consideration since the flow is generally turbulent, and the turbulent transport coefficients are usually at least an order of magnitude greater than their molecular counterparts. The phenomenological model employed for the turbulent transport coefficients in the present study is discussed in detail and compared with various other models.

The major assumptions employed in the development of the analysis are as follows:

1. The flow is either axisymmetric or two-dimensional.
2. The entire flow field is turbulent.
3. The gas obeys the perfect gas law.
4. The radial pressure gradient across the mixing zone is negligible.
5. Mass transfer due to thermal and pressure gradients is negligible.
6. The turbulent Prandtl and Lewis numbers of the gas are constant but may have values other than unity.
7. The particles may have a turbulent Lewis number different than that of the gas.
8. The velocity and temperature lags between the gas and particles are negligible.

9. The volume occupied by the particles is negligible compared to that of the gas.
10. The boundary layer between the mixed flow and the duct wall is neglected. The boundary layers in the rocket exhaust and the air stream at the initial point of contact, however, may be considered.
11. The turbulent boundary layer equations are valid within the mixing region.

1.3 Computer Program

The solution of the partial differential equations is obtained by first transforming the equations using the von Mises transformation, expressing the transformed equations in finite-difference form, and then solving the resulting finite-difference equations utilizing a computer program developed for the SRU 1107. Details of the computer program are presented in Reference 6. Consideration is given to the stability and convergence of the solution utilizing the criteria established by Wu (7).

The parameters which must be input to the program are as follows: (a) the initial conditions of velocity, temperature, pressure, and species concentrations of the two streams, (b) axial step-size, (c) duct geometry, (d) plume slip-line coordinates, (e) the turbulent Lewis and Prandtl numbers, and (f) computer control information as noted in Reference 6.

The output data comprises velocity, temperature, density, species concentration, and Mach number profiles within the mixing region at various axial locations; axial pressure distributions; and axial thrust acting on the duct.

The major limitations of the program are as follows: (a) the calculations are valid only in the temperature range 450R to 9950R, (b) the gaseous species which may be considered are limited to H_2O , CO , CO_2 , H_2 , O_2 , N_2 , HCl Al ; in addition, provision for one additional inert specie has been incorporated in the program, and (c) the particle specie which may be considered is Al_2O_3 .

Typical run time of the program is 10 minutes for a case involving a rocket nozzle radius of 0.21 ft, a duct radius of 0.56 ft, axial step-size of 0.01 ft, and a total axial distance of 6.0 ft.

2.0 DERIVATION OF THE GOVERNING EQUATIONS

2.1 Fundamental Equations

2.1.1 General

The derivation of the fundamental equations contained herein parallels the derivation of the steady-state turbulent boundary layer equations for gaseous systems. The effects of particles are included with the assumption that the particles and gas are in dynamic and thermal equilibrium, i.e., no velocity or temperature lags. The particles, therefore, are not treated as discrete droplets, but as chemical species which have properties corresponding to their respective liquid or solid phases. In determining the mass transfer of the particles resulting from turbulent mixing, it is assumed that the particles do not follow the random motions of the gas, but due to their inertia diffuse more slowly than the gas. The derivations that follow pertain to a two-dimensional flow model and later are expanded to include an axisymmetric flow system.

2.1.2 Global Continuity Equation

Consider a rectangular control volume located within the mixing region as shown in Figure 2. It is assumed that variations in the z-direction are negligible. The flow entering the elemental volume in the x-direction consists of a gas-particle mixture with an average axial velocity u . All the flow properties are assumed to be time-averaged values consistent with turbulent theory (8). The total mass flow entering the volume in the axial direction is

$$\rho_g u A_g + m_p u A_p$$

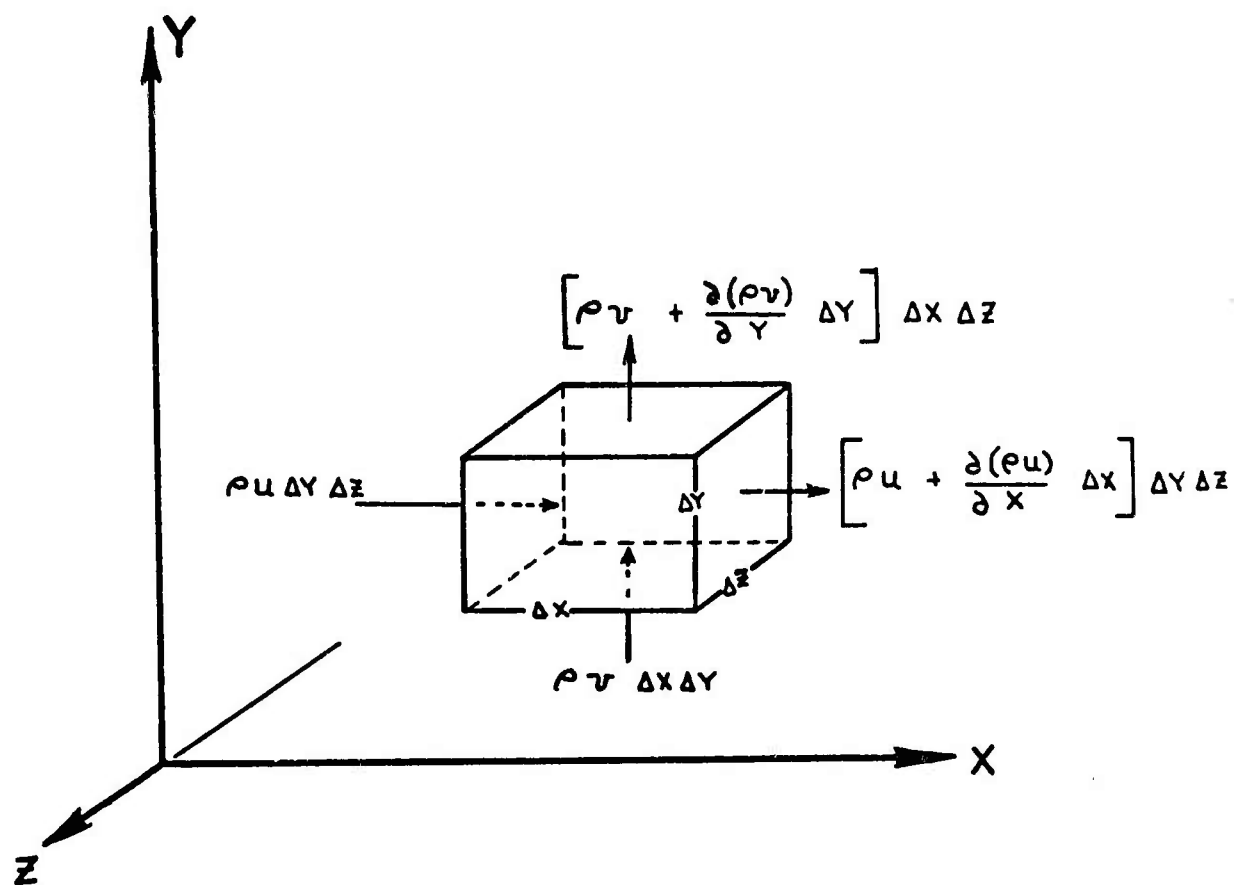


FIG. 2 Control Volume for Gas-Particle Mixtures

where ρ_g and ρ_p are the densities of the gas and particles, respectively, and A_g is the flow area occupied by the gas and A_p that by the particles. Then

$$A_g + A_p = \Delta y \Delta z$$

Assuming $A_g \gg A_p$ and letting ρ_p equal the weight of particles per unit volume of gas, yields

$$(\rho_g + \rho_p) u \Delta y \Delta z = \rho u \Delta y \Delta z$$

for the total axial mass flow into the volume where

$$\rho = \rho_g + \rho_p$$

The axial mass flow leaving the volume is given by

$$\rho u \Delta y \Delta z + \frac{\partial}{\partial x} (\rho u \Delta y \Delta z) \Delta x$$

The net change in axial mass flow is

$$\frac{\partial}{\partial x} (\rho u) \Delta x \Delta y \Delta z$$

Similarly in the y-direction the net change is

$$\frac{\partial}{\partial y} (\rho v) \Delta x \Delta y \Delta z$$

Conservation of mass yields

$$\frac{\partial}{\partial x} (\rho u) + \frac{\partial}{\partial y} (\rho v) = 0$$

(1)

for the steady-state global continuity equation.

2.1.3 Species Continuity Equations

The species continuity equations are derived from mass balances for each of the chemical species. The mass balance for component "i" of the mixture

flowing through the control volume consists of terms involving (a) the net mass change due to the flow entering and leaving the volume and (b) production (or depletion) of the species as a result of chemical reactions. The net transfer of component i in the x-direction due to the fluid motion is

$$- \frac{\partial}{\partial x} (\rho C_i u_i) \Delta x \Delta y \Delta z$$

where C_i is the mass fraction of component i (mass per unit mass of mixture) and U_i is its mean axial velocity. Similarly in the y-direction the net transfer is

$$- \frac{\partial}{\partial y} (\rho C_i v_i) \Delta x \Delta y \Delta z$$

The net rate of production of component i is given by

$$\dot{\omega}_i \Delta x \Delta y \Delta z$$

A mass balance for component i yields

$$\frac{\partial}{\partial x} (\rho C_i u_i) + \frac{\partial}{\partial y} (\rho C_i v_i) = \dot{\omega}_i \quad (2)$$

According to theory (9) the term $\rho C_i u_i$ may be expressed in terms of diffusion and convective fluxes. Thus

$$\rho C_i u_i = \dot{m}_{i,x} + \rho C_i u \quad (3)$$

where $\dot{m}_{i,x}$ is the diffusion flux resulting from concentration gradients and $\rho C_i u$ is the convective flux due to the bulk motion of the fluid. By bulk motion is meant the average macroscopic motion of the flow in which the axial bulk velocity u , is defined by

$$u \equiv \sum_{i=1}^n C_i u_i \quad (4)$$

where n is the total number of species comprising the mixture. For molecular diffusion the flux $\dot{m}_{i,x}$, may be expressed as

$$\dot{m}_{i,x} = -\rho D_i \frac{\partial C_i}{\partial x} \quad (5)$$

where D_i is the molecular diffusion coefficient for component i . For turbulent diffusion it has been shown theoretically (9) that a similar expression is valid with the molecular diffusivity replaced by its turbulent counterpart. Thus,

$$\dot{m}_{i,x} = -\rho \epsilon_{Di} \frac{\partial C_i}{\partial x} \quad (6)$$

where ϵ_{Di} is the eddy mass diffusivity of component i . Similarly in the y -direction

$$\dot{m}_{i,y} = -\rho \epsilon_{Di} \frac{\partial C_i}{\partial y} \quad (7)$$

Substitution of equations 3, 6, and 7 into equation 2 yields

$$\frac{\partial (\rho C_i u)}{\partial x} + \frac{\partial (\rho C_i v)}{\partial y} = \frac{\partial (\rho \epsilon_{Di} \frac{\partial C_i}{\partial x})}{\partial x} + \frac{\partial (\rho \epsilon_{Di} \frac{\partial C_i}{\partial y})}{\partial y} + \dot{w}_i$$

Expanding the above equation and substituting the global continuity equation yields

$$\rho u \frac{\partial C_i}{\partial x} + \rho v \frac{\partial C_i}{\partial y} = \frac{\partial (\rho \epsilon_{Di} \frac{\partial C_i}{\partial x})}{\partial x} + \frac{\partial (\rho \epsilon_{Di} \frac{\partial C_i}{\partial y})}{\partial y} + \dot{w}_i$$

It is generally assumed, based on an order of magnitude analysis, that the first term on the right side of the above equation is negligible. Thus the species continuity equation for component i becomes

$$\rho u \frac{\partial C_i}{\partial x} + \rho v \frac{\partial C_i}{\partial y} = \frac{\partial (\rho \epsilon_{Di} \frac{\partial C_i}{\partial y})}{\partial y} + \dot{w}_i$$

(8)

The net rates of production of the species are determined from a consideration of the chemical aspects of the problem. A detailed discussion of the determination of \dot{w}_i is presented in Section 4 herein.

The species continuity equation developed above is generally assumed valid for gaseous species, with the eddy mass diffusivities taken to be identical for all species. However, for particles further consideration must be given to the turbulent transport mechanism. The turbulent motions within a gas are usually conceived as macroscopic "lumps or eddies" of fluid moving randomly from one region to another. For gas-particle systems it seems unlikely that the particles, due to their inertia, are capable of following the random fluctuations of the gas. Consequently, the net transfer of particles from one region to another is likely to be considerably less than that of the gas. This difference in the turbulent transport of gas and particles may, in theory, be accomplished by employing different values for the eddy mass diffusivities of the two components. Thus, in the present analysis two values of the eddy mass diffusivity are employed - one for the gas and the other for the particles. A detailed discussion of the numerical values to be employed for the eddy mass diffusivities is presented in Section 3 herein.

2.1.4 Momentum Equations

The axial momentum equation is derived by equating the net change in axial momentum of the fluid flowing through the control volume to the net axial forces acting on the control boundaries. Thus

$$\frac{\partial(\rho u^2)}{\partial x} + \frac{\partial(\rho u^2)}{\partial y} = - \frac{\partial p}{\partial x} - \frac{\partial \tau_{xy}}{\partial y} \quad (9)$$

where τ_{xy} is the shear stress which, for turbulent flow, may be expressed in terms of the velocity gradient and the eddy momentum diffusivity. Thus

$$\tau_{xy} = -\rho \epsilon_v \frac{\partial u}{\partial y}$$

Expanding equation 9 and substituting equation 1 yields

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial y} \left(\rho \epsilon_v \frac{\partial u}{\partial y} \right) \quad (10)$$

Employing the usual boundary layer approximations and an order of magnitude analysis, the y-direction momentum equation reduces to

$$\frac{\partial p}{\partial y} = 0$$

2.1.5 Energy Equation

The energy equation is derived following the methods of Schlichting (10) and Vasiliu (3). For an observer moving with the flow the First Law of Thermodynamics may be stated as follows:

$$\left[\begin{array}{l} \text{rate of increase} \\ \text{of internal energy} \end{array} \right] = \left[\begin{array}{l} \text{rate of heat added} \\ \text{to the system} \end{array} \right] + \left[\begin{array}{l} \text{work done by the system} \\ \text{boundaries on the} \\ \text{surroundings} \end{array} \right]$$

The net change in internal energy of the gas-particle fluid flowing through the control volume may be expressed as

$$dE = \left(\rho u \frac{\partial e}{\partial x} + \rho v \frac{\partial e}{\partial y} \right) \Delta x \Delta y \Delta z \quad (11)$$

or

$$dE = \rho \frac{De}{Dt} \Delta x \Delta y \Delta z \quad (12)$$

where

$$\frac{D}{Dt} \equiv u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} \quad (13)$$

is the substantial derivative for steady-state and e is the internal energy.

From the definition of enthalpy, $h = e + P/\rho$, the substantive derivative of the internal energy may be written as

$$\frac{De}{Dt} = \frac{Dh}{Dt} - \frac{D(P/\rho)}{Dt} \quad (14)$$

where the enthalpy of the mixture is given by

$$h = \sum_{i=1}^n C_i h_i = \sum_{i=1}^n C_i \left(\int_0^T C_{p,i} dT + \Delta h_{f,i}^0 \right) \quad (15)$$

The heat of formation $\Delta h_{f,i}^0$, is evaluated at the reference temperature, 0°R . The substantive derivative of enthalpy is

$$\frac{Dh}{Dt} = \sum_{i=1}^n \left(C_i \frac{Dh_i}{Dt} + h_i \frac{DC_i}{Dt} \right) \quad (16)$$

Substitution of equations 14 and 16 into 12 yields

$$dE = \left[\rho \sum_{i=1}^n \left(C_i \frac{Dh_i}{Dt} + h_i \frac{DC_i}{Dt} \right) - \rho \frac{D(P/\rho)}{Dt} \right] \Delta x \Delta y \Delta z$$

or

$$dE = \left[\rho \sum_{i=1}^n \left(C_i \frac{dh_i}{dT} \frac{DT}{Dt} + h_i \frac{DC_i}{Dt} \right) - \rho \frac{D(P/\rho)}{Dt} \right] \Delta x \Delta y \Delta z$$

But

$$\frac{dh_i}{dT} = C_{p,i}$$

and

$$\sum_{i=1}^n C_i C_{p,i} = C_p$$

Thus, the net change of the gas-particle internal energy becomes

$$dE = \left[\rho c_p \frac{DT}{Dt} + \rho \sum_{i=1}^n h_i \frac{DC_i}{Dt} - \rho \frac{D(P/\rho)}{Dt} \right] \Delta x \Delta y \Delta z \quad (17)$$

The heat transferred or added to the control volume is assumed to be that resulting from the following processes: (a) conduction and diffusion, (b) frictional heating, and (c) chemical reactions. The net heat transfer in the y-direction due to conduction and diffusion may be expressed as

$$dQ_y = \frac{\partial}{\partial y} \left[\left(\rho c_p \epsilon_H \frac{\partial T}{\partial y} \right) - \sum_{i=1}^n \rho v_{xi} c_i h_i \right] \Delta x \Delta y \Delta z$$

where ϵ_H is the eddy diffusivity for heat and v_{xi} is the y-direction diffusion velocity of component i defined by

$$v_{xi} = - \frac{\epsilon_{D_i}}{c_i} \frac{\partial c_i}{\partial y}$$

Thus the net heat transfer term becomes

$$dQ_y = \left[\frac{\partial}{\partial y} \left(\rho c_p \epsilon_H \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial y} \sum_{i=1}^n \rho \epsilon_{D_i} h_i \frac{\partial c_i}{\partial y} \right] \Delta x \Delta y \Delta z$$

Expanding the second term in the above equation and substituting the species continuity equation yields

$$dQ_y = \left[\frac{\partial}{\partial y} \left(\rho c_p \epsilon_H \frac{\partial T}{\partial y} \right) + \sum_{i=1}^n \left(\rho \frac{DC_i}{Dt} - \dot{\omega}_i \right) h_i + \sum_{i=1}^n \rho \epsilon_{D_i} c_{xi} \frac{\partial c_i}{\partial y} \frac{\partial T}{\partial y} \right] \Delta x \Delta y \Delta z \quad (18)$$

The frictional heating term is assumed identical to that generally employed in boundary layer analyses. Thus

$$dQ_f = \rho \epsilon_v \left(\frac{\partial u}{\partial y} \right)^2 \Delta x \Delta y \Delta z \quad (19)$$

The heat release due to chemical reaction is accounted for by including the heat of formation in the enthalpy term for each component. Thus the enthalpy of the mixture is

$$h = \sum_{i=1}^n C_i \left[\int_0^T C_{p,i} dT + \Delta h_{f,i}^{\circ} \right] \quad (20)$$

The net rate of work done by the control volume surface forces on the surroundings is shown by Schlichting to be given by

$$dW = - \rho \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \quad (21)$$

Employing equations 17, 18, 19 and 21 the energy equation becomes

$$\begin{aligned} \rho C_p \frac{DT}{Dt} = \frac{\partial}{\partial y} \left(\rho C_p \epsilon_H \frac{\partial T}{\partial y} \right) + \rho \frac{D(P/\rho)}{Dt} - \rho \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \\ + \rho \epsilon_v \left(\frac{\partial u}{\partial y} \right)^2 - \sum_{i=1}^n \dot{w}_i h_i + \sum_{i=1}^n \rho \epsilon_{D,i} C_{p,i} \frac{\partial C_i}{\partial y} \frac{\partial T}{\partial y} \end{aligned}$$

The second term on the right side of the above equation may be simplified utilizing the global continuity equation. Thus

$$\rho \frac{D(P/\rho)}{Dt} = \frac{DP}{Dt} - \frac{P}{\rho} \frac{D\rho}{Dt} = \frac{DP}{Dt} + P \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right)$$

Thus the energy equations becomes

$$\begin{aligned} \rho C_p \left(u \frac{\partial T}{\partial y} + v \frac{\partial T}{\partial y} \right) = \frac{\partial}{\partial y} \left(\rho C_p \epsilon_H \frac{\partial T}{\partial y} \right) + u \frac{\partial P}{\partial x} \\ + \rho \epsilon_v \left(\frac{\partial u}{\partial y} \right)^2 + \sum_{i=1}^n \left(\rho \epsilon_{D,i} C_{p,i} \frac{\partial C_i}{\partial y} \frac{\partial T}{\partial y} \right) - \sum_{i=1}^n (\dot{w}_i h_i) \end{aligned} \quad (22)$$

2.1.6 Equation of State

The equation of state for the gas-particle mixture is developed based on the assumptions that (a) the gas obeys the perfect gas law and (b) the volume occupied by the particles is negligible. Consider a unit mass of mixture comprising a mass C_{ig} , of gaseous products, and a mass C_{ip} of particles. The total volume of the mixture is given by

$$\frac{C_{ip}}{m_p} + \frac{C_{ig}}{\rho_g} = \frac{1}{\rho}$$

where m_p and ρ_g are the densities of the particles and gas, respectively. Assuming that the partial volume occupied by the particles is negligible the expression for the total volume becomes

$$\frac{1}{\rho} = \frac{C_{ig}}{\rho_g} \quad (23)$$

The equation of state for the gaseous phase is

$$P = \rho_g R_g T \quad (24)$$

where R_g is the gas constant for the gaseous components, which may be expressed in terms of the universal gas constant and the gas molecular weight. Thus

$$R_g = \frac{R_u}{W_g} \quad (25)$$

where

$$W_g = \left[\sum_{i=1}^{n_g} C_i / W_i \right]^{-1} \sum_{i=1}^{n_g} C_i \quad (26)$$

The summation is performed only for the gaseous species - n_g being the total number of gaseous components. The total density of the mixture is obtained from equations 23 - 26. Thus

$$\rho = \frac{P}{R_u T} \left[\sum_{i=1}^{n_g} C_i / W_i \right]^{-1} \quad (27)$$

2.1.7 Alternate Forms of the Fundamental Equations

The governing equations may be expressed in alternate forms employing dimensionless ratios involving the eddy diffusivities. By definition the turbulent Lewis and Prandtl numbers are

$$Le_i \equiv \frac{\epsilon_{D_i}}{\epsilon_H} \quad (28)$$

$$Pr \equiv \frac{\epsilon_V}{\epsilon_H} \quad (29)$$

where ρ and C_p are the density and specific heat of the gas-particle mixture, respectively. Employing these dimensionless numbers the fundamental equations, in axisymmetric coordinates, become

Global Continuity:

$$\frac{\partial}{\partial x}(\rho u r^\delta) + \frac{\partial}{\partial r}(\rho v r^\delta) = 0 \quad (30)$$

Species Continuity:

$$\rho u \frac{\partial C_i}{\partial x} + \rho v \frac{\partial C_i}{\partial r} = \frac{1}{r^\delta} \frac{\partial}{\partial r} \left(r^\delta \rho \epsilon_v \frac{Le_i}{Pr} \frac{\partial C_i}{\partial r} \right) + \dot{\omega}_i \quad (31)$$

Momentum Equation:

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial r} = - \frac{\partial P}{\partial x} + \frac{1}{r^\delta} \frac{\partial}{\partial r} \left(\rho \epsilon_v r^\delta \frac{\partial u}{\partial r} \right) \quad (32)$$

Energy Equation:

$$c_p \left(\rho u \frac{\partial T}{\partial x} + \rho v \frac{\partial T}{\partial r} \right) = u \frac{\partial \rho}{\partial x} + \rho \epsilon_v \left(\frac{\partial u}{\partial r} \right)^2 + \frac{1}{r^\delta} \frac{\partial}{\partial r} \left(c_p r^\delta \rho \frac{\epsilon_v}{Pr} \frac{\partial T}{\partial r} \right) + \frac{\rho \epsilon_v}{Pr} \sum_{i=1}^{n_g} \left(c_{p,i} Le_i \frac{\partial T}{\partial r} \frac{\partial C_i}{\partial r} \right) - \sum_{i=1}^{n_g} (\dot{\omega}_i h_i) \quad (33)$$

Equation of State:

$$\rho = \frac{P}{R_u T} \left[\sum_{i=1}^{n_g} C_i / W_i \right]^{-1} \quad (34)$$

The exponent for the terms r^δ , makes the equations applicable for either two-dimensional ($\delta=0$) or axisymmetric flow ($\delta=1$). A system of $(4 + n)$ simultaneous equations are therefore available for determining the unknown parameters $u, v, T, \rho, C_i, \dot{\omega}_i$, and ϵ_v . Thus, a solution may be obtained if appropriate expressions for the parameters $\dot{\omega}_i$ and ϵ_v are available in terms of the remaining unknown parameters. Such expressions will be developed later herein; however, for the present these parameters will be retained in their present form with the assumption that suitable expressions are available.

2.2 Transformation of the Fundamental Equations

In obtaining a solution it is convenient to transform the above equations utilizing the von Mises transformation. Instead of the axisymmetric coordinates x and r (or the cartesian coordinates x and y) the stream function ψ , is introduced as one of the independent variables replacing the r -coordinate. The x -coordinate is not affected by the transformation. By definition the stream function is

$$\left(\frac{\partial \psi}{\partial r} \right)_x = \rho u r^\delta \quad (35)$$

$$\left(\frac{\partial \psi}{\partial x} \right)_r = -\rho v r^\delta \quad (36)$$

which when substituted into the global continuity equation (equation 30) yields

$$\frac{\partial}{\partial x} \left(\frac{\partial \psi}{\partial r} \right) - \frac{\partial}{\partial r} \left(\frac{\partial \psi}{\partial x} \right) = 0$$

Thus, the continuity equation is satisfied identically since

$$\frac{\partial^2 \psi}{\partial x \partial r} = \frac{\partial^2 \psi}{\partial r \partial x}$$

The transformation of the remaining equations is accomplished by noting, from the chain rule of the calculus, that

$$\begin{aligned} \left(\frac{\partial f}{\partial r} \right)_x &= \left(\frac{\partial f}{\partial \psi} \right)_x \left(\frac{\partial \psi}{\partial r} \right)_x \\ &= \rho u r^5 \left(\frac{\partial f}{\partial \psi} \right)_x \end{aligned}$$

and

$$\begin{aligned} \left(\frac{\partial f}{\partial x} \right)_r &= \left(\frac{\partial f}{\partial \psi} \right)_x \left(\frac{\partial \psi}{\partial x} \right)_r + \left(\frac{\partial f}{\partial x} \right)_\psi \\ &= -\rho v r^5 \left(\frac{\partial f}{\partial \psi} \right)_x + \left(\frac{\partial f}{\partial x} \right)_\psi \end{aligned}$$

where f represents any of the dependent variables. Hence, the transformed equations become

Species Continuity:

$$\frac{\partial C_i}{\partial x} = \frac{\partial}{\partial \psi} \left(\frac{\epsilon_i L e_i}{Pr} \rho^2 u r^{15} \frac{\partial C_i}{\partial \psi} \right) + \frac{\dot{w}_i}{\rho u} \quad (37)$$

Momentum Equation:

$$\frac{\partial u}{\partial x} = -\frac{1}{\rho u} \frac{dP}{dx} + \frac{\partial}{\partial \psi} \left(\epsilon_i \rho^2 u r^{15} \frac{\partial u}{\partial \psi} \right) \quad (38)$$

Energy Equation:

$$\begin{aligned} \frac{\partial T}{\partial x} = & \frac{1}{\rho C_p} \frac{dP}{dx} + \frac{\epsilon_v \rho^2 u r^{2\delta}}{C_p} \left(\frac{\partial u}{\partial \psi} \right)^2 + \frac{1}{C_p} \frac{\partial}{\partial \psi} \left(C_p \frac{\epsilon_v \rho^2 u r^{2\delta}}{Pr} \frac{\partial T}{\partial \psi} \right) \\ & + \frac{\rho^2 u r^{2\delta} \epsilon_v}{C_p Pr} \sum_{i=1}^n \left(C_{p,i} Le_i \frac{\partial T}{\partial \psi} \frac{\partial C_i}{\partial \psi} \right) - \sum_{i=1}^n (\dot{w}_i h_i) \end{aligned} \quad (39)$$

In order to obtain the solution in the physical plane (x, r), it is necessary to transform the ψ -coordinate to the r-coordinate. From the definition of the stream function, equation 35, the r-value corresponding to any ψ -value is given by

$$r = \left(r_0^{\delta+1} + (\delta+1) \int_0^\psi \frac{d\psi}{\rho u} \right)^{1/(\delta+1)} \quad (40)$$

where r is the r-value corresponding to the reference streamline, $\psi = 0$.

Shown in Figure 3 is the orientation of the x-r coordinate system and the location of streamlines for a typical problem.

2.3 Development of the Finite-Difference Equations

The finite difference technique of solving differential equations consists of replacing the partial derivatives by finite difference ratios, which is equivalent to physically replacing the continuous flow field by a network of finite elements. Consider the flow field illustrated in Figure 4 and assume it to be divided into a number of elements having grid sizes of Δx and $\Delta \psi$. If, in addition, it is assumed that values of all the flow properties are known along the vertical line designated m, referred to as "front m", it is then possible to calculate the flow properties at front m + 1 in the following manner.

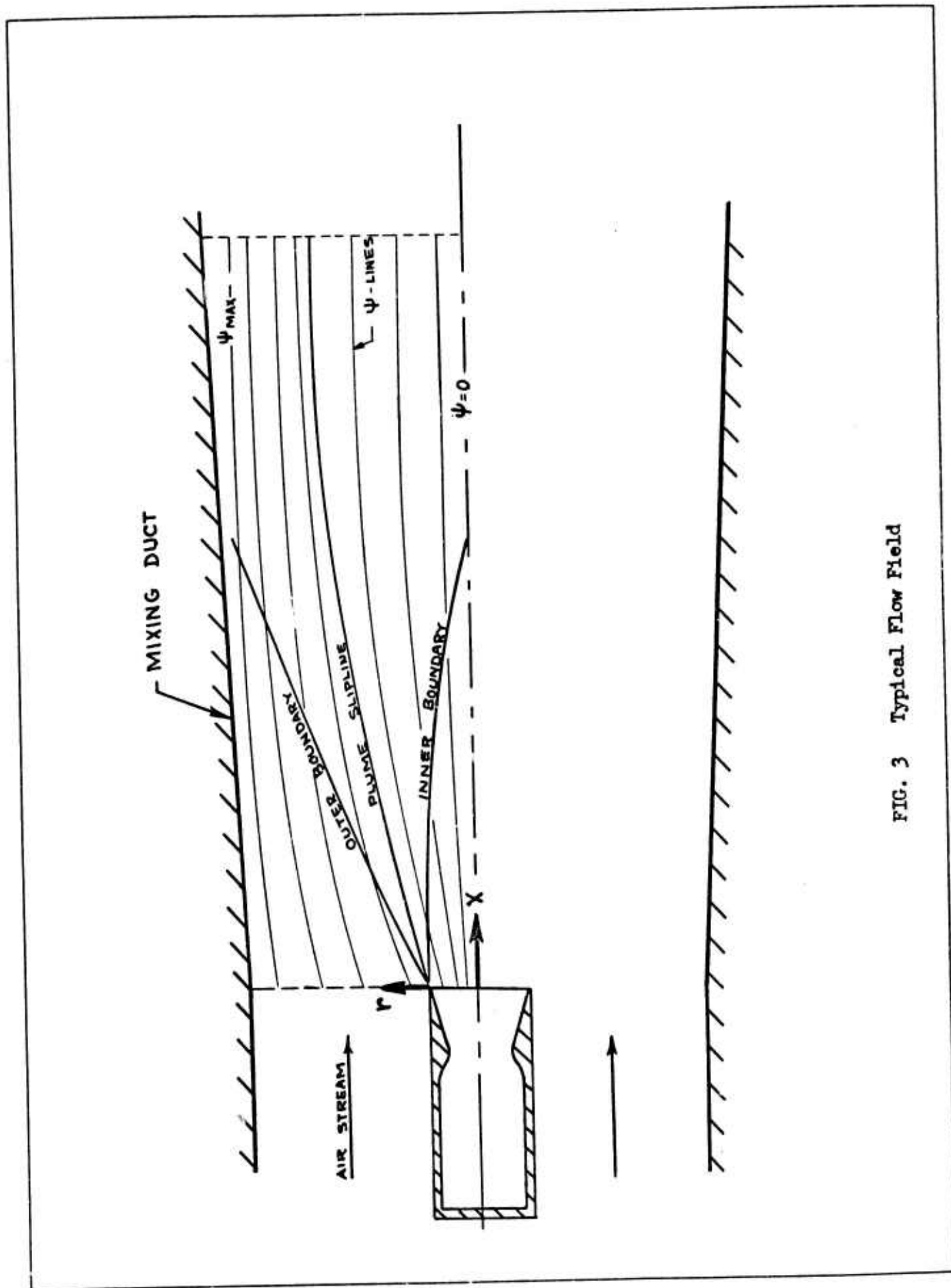


FIG. 3 Typical Flow Field

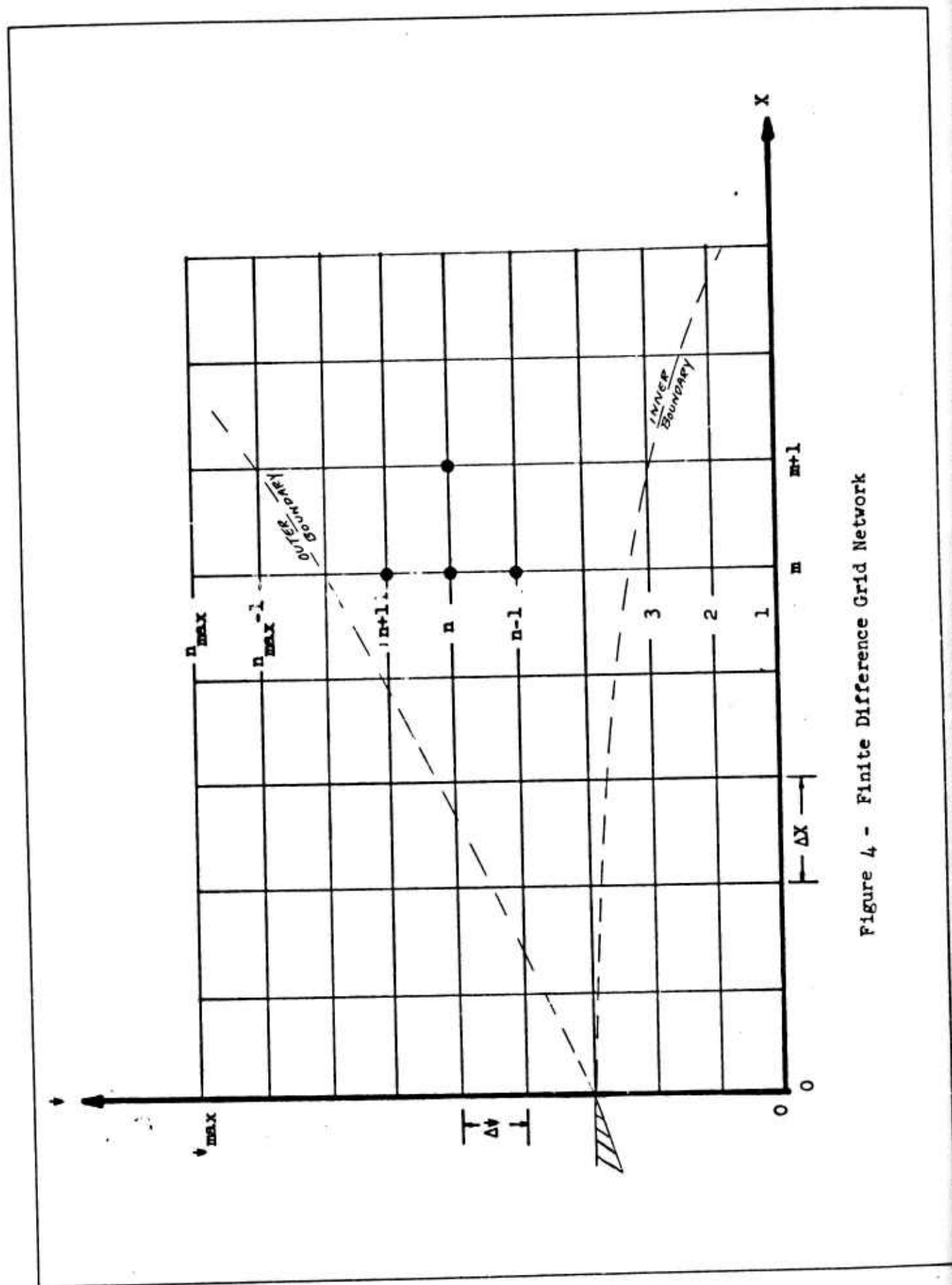


Figure 4 - Finite Difference Grid Network

REV LTR

U3 4288-2000 REV. 6/64

BOEING

NO.

D2-36251-1

SH.

32

For the grid point denoted $(m + 1, n)$ a Taylor series expansion of the function f in terms of x -derivatives yields

$$f_{m+1,n} = f_{m,n} + \Delta x \left(\frac{\partial f}{\partial x} \right)_{m,n} + \frac{\Delta x^2}{2!} \left(\frac{\partial^2 f}{\partial x^2} \right)_{m,n} + \dots$$

where f may be taken to be any dependent variable, e.g., u , t , e , etc. If Δx is taken sufficiently small the terms of higher order than unity may be neglected. thus the x -derivative becomes:

$$\left(\frac{\partial f}{\partial x} \right)_{m,n} = \frac{f_{m+1,n} - f_{m,n}}{\Delta x}$$

The above equation is termed the forward-difference approximation for the x -derivative. In a similar manner backward - difference and average - difference approximations may be derived (7). The resulting backward- and average-difference approximations are, respectively

$$\left(\frac{\partial f}{\partial x} \right)_{m,n} = \frac{f_{m,n} - f_{m-1,n}}{\Delta x}$$

$$\left(\frac{\partial f}{\partial x} \right)_{m,n} = \frac{f_{m+1,n} - f_{m-1,n}}{2 \Delta x}$$

Following the analysis of Wu (7) the forward-difference approximations are used herein for the x -derivatives, and the average-difference approximation for the ψ -derivatives. The selection of these approximations is based on physical and stability considerations as discussed by Wu. In the present analysis, the following approximations are employed for the

derivatives

$$\left(\frac{\partial f}{\partial x}\right)_{m,n} = \frac{f_{m+1,n} - f_{m,n}}{\Delta x} \quad (41)$$

$$\left(\frac{\partial f}{\partial \psi}\right)_{m,n} = \frac{f_{m,n+1} - f_{m,n-1}}{2 \Delta \psi} \quad (42)$$

The second derivative with respect to ψ is given by the finite-difference approximation

$$\frac{\partial^2 f}{\partial \psi^2} = \frac{f_{m,n+1} - 2f_{m,n} + f_{m,n-1}}{\Delta \psi^2} \quad (43)$$

The product terms appearing in the differential equations are approximated by

$$\frac{\partial f}{\partial \psi} \cdot \frac{\partial g}{\partial \psi} = \frac{(f_{m,n+1} - f_{m,n-1}) \cdot (g_{m,n+1} - g_{m,n-1})}{4 \Delta \psi^2} \quad (44)$$

Substituting the above expressions for the derivatives appearing in equations 37-39 and rearranging terms, yields

Species Continuity:

$$\begin{aligned} C_{i,m+1,n} = & C_{i,m,n} + \frac{\Delta x}{\Delta \psi^2} \frac{Le_i}{Pr} (\rho^2 u r^{2\delta} \epsilon_v)_{m,n} (C_{i,m,n+1} - 2C_{i,m,n} + C_{i,m,n-1}) \\ & + \frac{\Delta x}{4 \Delta \psi^2} \frac{Le_i}{Pr} (C_{i,m,n+1} - C_{i,m,n-1}) \left[(\rho \epsilon_v)_{m,n} \{ (\rho u r^{2\delta})_{m,n+1} - (\rho u r^{2\delta})_{m,n-1} \} \right. \\ & \left. + (\rho u r^{2\delta})_{m,n} \{ (\rho \epsilon_v)_{m,n+1} - (\rho \epsilon_v)_{m,n-1} \} \right] + \left(\frac{\dot{w}_i \Delta x}{\rho u} \right)_{m,n} \end{aligned} \quad (45)$$

Momentum Equation:

$$\begin{aligned} u_{m+1,n} = & u_{m,n} - \left(\frac{\Delta x g}{\rho u} \right)_{m,n} \left(\frac{dp}{dx} \right)_m + \frac{\Delta x}{\Delta \psi^2} (\rho^2 u r^{2\delta} \epsilon_v)_{m,n} (u_{m,n+1} - 2u_{m,n} + u_{m,n-1}) \\ & + \frac{\Delta x}{4 \Delta \psi^2} \left[(\rho \epsilon_v)_{m,n} \{ (\rho u r^{2\delta})_{m,n+1} - (\rho u r^{2\delta})_{m,n-1} \} \right. \\ & \left. + (\rho u r^{2\delta})_{m,n} \{ (\rho \epsilon_v)_{m,n+1} - (\rho \epsilon_v)_{m,n-1} \} \right] (u_{m,n+1} - u_{m,n-1}) \end{aligned} \quad (46)$$

Energy Equation:

$$\begin{aligned}
 T_{m+1,n} = & T_{m,n} + \left(\frac{\Delta X}{J \rho C_p} \frac{dP}{dx} \right)_{m,n} + \frac{\Delta X}{4 \Delta \psi^2 P_r} (\rho u r^{2\delta})_{m,n} \left[\frac{(u_{m,n+1} - u_{m,n-1})^2}{J g (C_p)_{m,n}} P_r (\rho E_v)_{m,n} \right. \\
 & + 4 (\rho E_v)_{m,n} (T_{m,n+1} - 2T_{m,n} + T_{m,n-1}) + \{ (\rho E_v)_{m,n+1} - (\rho E_v)_{m,n-1} \} (T_{m,n+1} - T_{m,n-1}) \\
 & + \frac{\Delta X}{4 \Delta \psi^2 P_r} \left(\frac{\rho E_v}{C_p} \right)_{m,n} \left[(\rho u r^{2\delta} C_p)_{m,n+1} - (\rho u r^{2\delta} C_p)_{m,n-1} \right] (T_{m,n+1} - T_{m,n-1}) \quad (47) \\
 & + \frac{\Delta X}{4 \Delta \psi^2 P_r} \left(\frac{\rho^2 u r^{2\delta} E_v}{C_p} \right)_{m,n} \sum_i C_{p,i} \log \left(\frac{C_{i,m,n+1}}{C_{i,m,n-1}} \right) (T_{m,n+1} - T_{m,n-1}) - \frac{\Delta X}{(\rho u C_p)_{m,n}} \sum_i (\dot{w}_i h_i)_{m,n}
 \end{aligned}$$

As can be seen, the above equations form a set of algebraic expressions in which the dependent variable at the $m + 1$ front is expressed explicitly in terms of the known properties at front m . The calculation of the flow properties throughout the grid network may then be carried out by successively applying the equations to each front. However, it is first necessary that the boundary conditions be specified.

2.4 Initial and Boundary Conditions

In starting the calculations for a specific problem, it is necessary that the properties along the initial front be known at each grid point - these properties are termed the initial conditions. The properties may be constant or variable dependent on the problem under consideration. For example, it may be desired to assume constant properties for the rocket exhaust jet except in the immediate neighborhood of the nozzle wall where the jet boundary layer exists. Similarly, the flow properties in the outer stream along the initial front may be variable corresponding to those existing within a boundary layer. In other problems, it may be desired to start the calculations at a point downstream where the mixing region has partially developed and the calculations continued for further development of the mixing region. For any of these situations, it is required that the properties be known at each grid point along the initial front before the calculations can begin.

Once the initial conditions are specified the flow properties at each grid point along the second front may be calculated except at the grid points corresponding to $\psi = 0$ and $\psi = \psi_{\max}$. These streamlines are the boundaries of the grid network within which the mixing region lies. In order to obtain a complete solution for a given flow problem, it is necessary that the flow properties along the boundaries be known. These properties are termed boundary conditions. It is the usual practice in free jet problems to assume the flow to be inviscid in the areas outside the mixing region. The boundary conditions for such problems are therefore specified accordingly.

For problems in which the secondary flow is contained within a duct, however, three regimes of flow may be encountered each dictating different boundary conditions. For example, consider the flow field shown in Figure 5, illustrating the different flow regimes. The first regime begins at the exit plane of the nozzle and ends at the point where the inner edge of the mixing zone reaches the jet centerline. The boundary conditions for this regime may be taken to correspond to those for inviscid flow. The flow field, therefore, resembles that of free jet problems.

In the second regime, the mixing region widens until at some point the outer edge of the mixing zone reaches the duct wall, whereupon the third regime commences. The technique for determining the outer boundary condition in the second regime is identical to that in the first, i.e., the secondary flow outside the mixing region is assumed to be inviscid. The inner boundary conditions for the second and third flow regimes may be determined from the physical characteristics of the flow. Along the jet

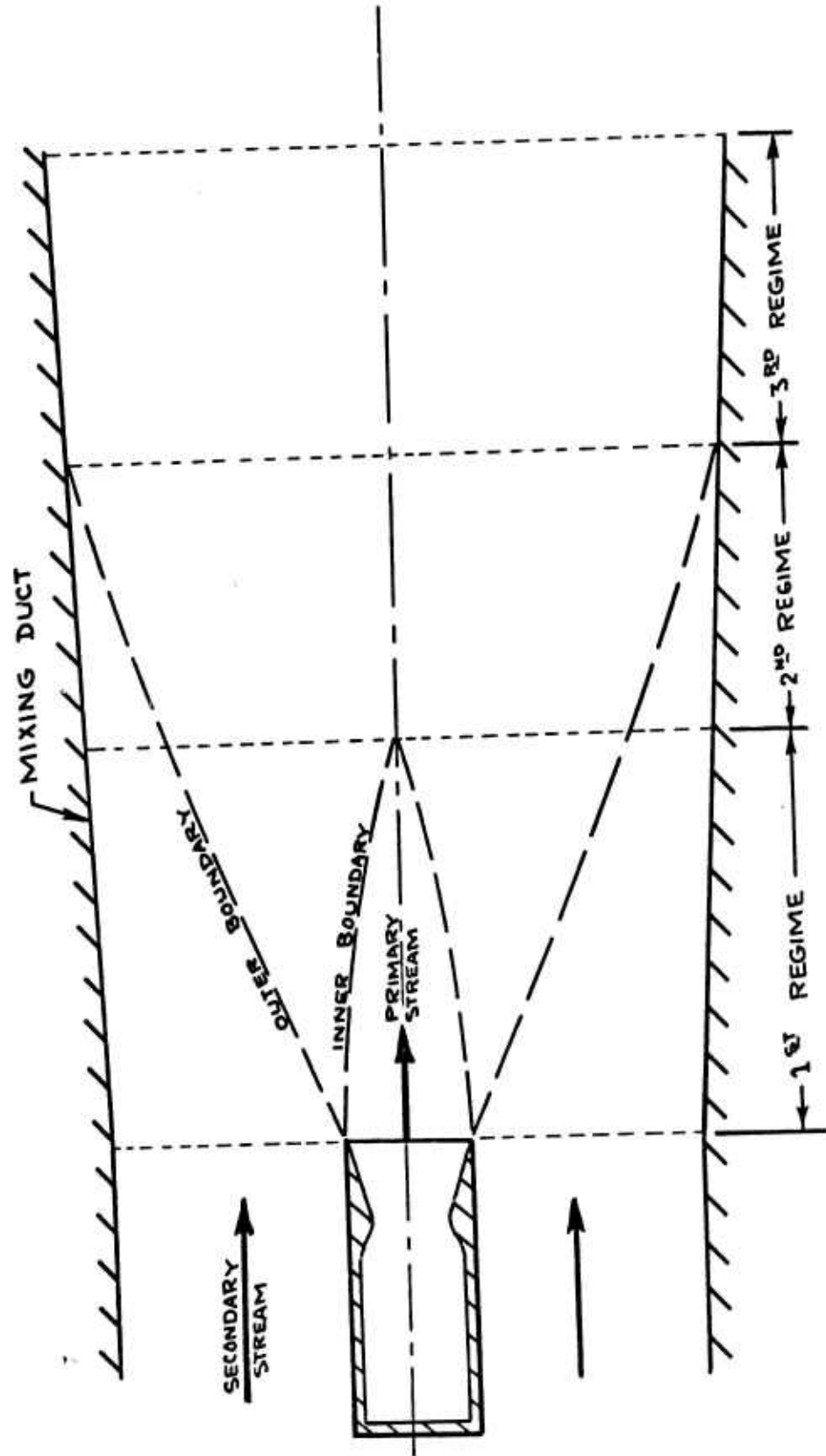


FIG. 5 Flow Regimes for Confined Mixing

centerline, the flow properties change as a result of the mixing process. However, from symmetry it may be concluded that on the centerline, the radial gradients of all properties must be zero. This condition may therefore be employed as the inner boundary condition for the second regime.

In the third regime, the inviscid outer boundary assumption is no longer valid since the mixing region extends across the entire duct. However, the outer boundary condition for this regime may be established by assuming that the heat transfer and shear stress at the duct wall are negligible. Thus, from the rate equations for shear stress and heat transfer it is seen that

$$\left(\frac{\partial u}{\partial \psi} \right)_{wall} = 0$$

$$\left(\frac{\partial T}{\partial \psi} \right)_{wall} = 0$$

In addition, since mass cannot be transferred across the duct wall, we have

$$\left(\frac{\partial C_i}{\partial \psi} \right)_{wall} = 0$$

Thus, the inner and outer boundary conditions for the third regime are identical. Summarizing, the boundary conditions for the first flow regime may be expressed in the following form:

$$C_{i,m+1} = C_{i,m} + \left(\frac{\dot{w}_i \Delta x}{\rho u} \right)_m \quad (48)$$

$$U_{m+1} = U_m + \left(\frac{\Delta X g}{\rho U} \right)_m \left(\frac{dP}{dX} \right)_m \quad (49)$$

$$T_{m+1} = T_m + \left(\frac{\Delta X}{\mathcal{F} \rho C_p} \frac{dP}{dX} \right)_m - \left(\frac{\Delta X}{\rho U C_p} \sum_{j=1}^n \dot{w}_j h_j \right)_m \quad (50)$$

The above equations are applicable for determining both the inner and outer boundary conditions. In the second regime, the boundary conditions for the inviscid region are as above; whereas, the inner boundary conditions are given by

$$\left(\frac{\partial C_i}{\partial \psi} \right)_{\psi=0} = \left(\frac{\partial U}{\partial \psi} \right)_{\psi=0} = \left(\frac{\partial T}{\partial \psi} \right)_{\psi=0} = 0 \quad (51)$$

For the third regime, the boundary conditions are expressed in the form

$$\left(\frac{\partial C_i}{\partial \psi} \right)_{\psi=0} = \left(\frac{\partial C_i}{\partial \psi} \right)_{\psi=\psi_{\max}} = 0 \quad (52)$$

$$\left(\frac{\partial U}{\partial \psi} \right)_{\psi=0} = \left(\frac{\partial U}{\partial \psi} \right)_{\psi=\psi_{\max}} = 0$$

$$\left(\frac{\partial T}{\partial \psi} \right)_{\psi=0} = \left(\frac{\partial T}{\partial \psi} \right)_{\psi=\psi_{\max}} = 0$$

It should be noted that in the above discussion it was assumed that the inner edge of the mixing zone reached the duct centerline before the outer

edge reached the duct wall. In certain situations, however, these conditions may be reversed, the outer edge reaching the duct wall prior to the inner edge reaching the centerline, in which case the aforementioned boundary conditions are altered accordingly.

REV LTR

U3 4288-2000 REV. 6/64

BOEING

NO. D2-36251-1

SH.

40

3.0 TURBULENT TRANSPORT COEFFICIENTS

3.1 General

In order to obtain a solution of the finite-difference equations, it is necessary that expressions for the eddy diffusivities be established in terms of known quantities. Unfortunately the eddy diffusivities are not fluid properties, as are their molecular counterparts, but parameters dependent upon the fluid motion, e.g., level of turbulence, velocity gradient, density gradient, etc. Since turbulent flow is characterized by random fluctuations, the derivation of accurate theoretical expressions for the eddy diffusivities requires a statistical analysis such as that initiated by Taylor (11). However, the statistical approach at the present time has not proven practical (12). Consequently, one must rely on the semi-empirical approach for determining expressions relating the eddy diffusivities to the flow properties.

3.2 Incompressible Theory

Most of the semi-empirical theories to date are extensions or modifications of Prandtl's mixing length concept. From Prandtl's initial hypothesis, it was found that the shear stress for incompressible turbulent flow can be determined from

$$\tau = \rho \ell^2 \left(\frac{\partial u}{\partial y} \right)^2 \quad (53)$$

where ℓ is Prandtl's mixing length parameter, assumed to be independent of the y-coordinate (1, 10). Broussinesq had earlier postulated that, in analogy with the shear stress law for laminar flow, the shear stress for

turbulent flow could be expressed in the form:

$$\tau = \rho \epsilon_v \frac{\partial u}{\partial y} \quad (54)$$

where ϵ_v is the eddy diffusivity of momentum, often referred to as eddy viscosity, being analogous to molecular kinematic viscosity. From equations 53 and 54 it is found that

$$\epsilon_v = l^2 \frac{\partial u}{\partial y} \quad (55)$$

In an attempt to obtain more realistic values of ϵ_v when $\partial u / \partial y$ approaches zero, Prandtl modified the above expression; however, the resulting expression is difficult to use.

A simpler expression for ϵ_v was subsequently developed by Prandtl based on the experimental data of Reichardt. Assuming that ϵ_v was constant across the mixing zone, i.e., independent of the y-coordinate, Prandtl postulated that for free jets

$$\epsilon_v = k b (U_{\max} - U_{\min}) \quad (56)$$

where k is an empirical constant, b is the width of the mixing region, and $(U_{\max} - U_{\min})$ is the difference between the maximum and minimum velocities in the mixing region. Furthermore, it was found that

$$b = c x$$

so that Eqn. 56 becomes

$$\epsilon_v = K X (U_{max} - U_{min}) \quad (57)$$

where the product kc , is replaced by another proportionately constant K . From Reichardt's experiments of incompressible free jets ($U_{min} = 0$), it was determined that the width of the mixing zone varied according to the relationship

$$b = 0.098 X \quad (58)$$

The resulting expression for ϵ_v which best correlated the data was

$$\epsilon_v = 0.00137 X U_{max} \quad (59)$$

An extension of the above expression is often employed for the case of two moving streams, thus

$$\epsilon_v = 0.00137 X (U_{max} - U_{min}) \quad (60)$$

An alternate form of Equation 57 is frequently used in correlating experiment and theory in terms of the similarity parameter, thus

$$\epsilon_v = \frac{U_{max} + U_{min}}{4 \sigma^2} X \quad (61)$$

where σ the similarity parameter, is a constant to be determined from experimental data. Physically, σ is a parameter related to the spreading rate of the mixing region - larger values of σ corresponding

to smaller spreading rates. From the data of Reichardt, employed in obtaining Equation 60, the corresponding value of σ was determined to be 13.5.

Still another expression for ϵ_v is that given by Pai (12), which in its most general form is

$$\epsilon_v = \epsilon_o (C + x/L)^n \quad (62)$$

where ϵ_o is referred to as Reichardt's coefficient, C is a constant which accounts for a virtual origin of the mixing region, L is a reference length, and n is a number having a value between 0 and 1 depending on the degree of mixing. The constant C may be assumed to account for the boundary layers that build up in the two streams prior to the point of initial contact ($x = 0$) which produce, in effect, a mixing region of finite width at $x = 0$. For the case where $C = 0$ and $n = 1$, as generally is assumed, Equation 62 becomes

$$\epsilon_v = \epsilon_o x/L \quad (63)$$

Equating Equations 61 and 63 and solving for ϵ_o yields

$$\epsilon_o = \frac{L (u_{max} + u_{min})}{4 \sigma^2} \quad (64)$$

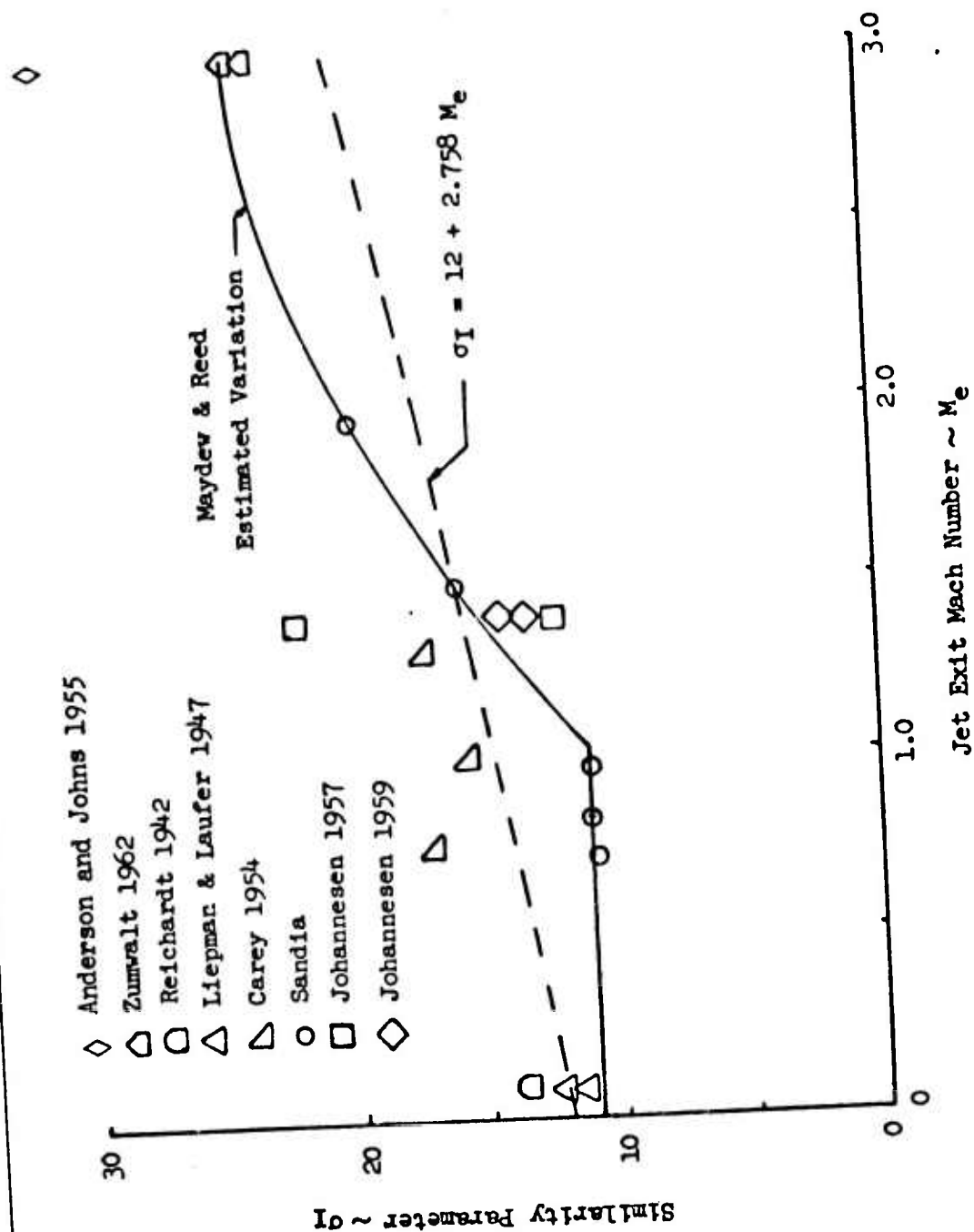
From extensive data of free jets exhausting into a quiescent atmosphere, the value of σ for jet Mach numbers less than unity has been relatively well established as being $\sigma = 12$. For cases where the receiving medium is

also moving the value of σ is found to depend on the velocity and density ratio between the two streams as discussed below.

3.3 Compressible Theory

In the development of the ϵ_v expressions for incompressible flow it was assumed that ϵ_v was constant across the mixing zone, an assumption which has been substantiated by extensive experimental data. However, for compressible flow involving large temperature and velocity differences between the two streams, it has been postulated that ϵ_v is dependent not only on the x-coordinate, but the y-coordinate as well (13). Several mathematical models for the compressible eddy viscosity have been proposed by various investigators (14, 15, 16), which for the most part are more complicated and consequently more difficult to use than the incompressible models. In addition, since only limited experimental data are available, none of the proposed compressible models has been satisfactorily verified. Consequently it has been the general practice to employ the incompressible model for ϵ_v and adjust the empirical constants to account for the effects of compressibility.

The most frequently used parameter in correlating experiment and theory is the similarity parameter σ , defined in Equation 61. Experimentally determined values of σ for free jets exhausting into a quiescent atmosphere have been obtained for jet Mach numbers ranging up to approximately 3.0 (15). Figure 6 illustrates the variation of σ with jet Mach number for a jet exhausting into still air. The solid line faired through the experimental data was originally presented in Reference 15. The dashed line illustrates the theoretical variation of σ with Mach number as given by Korst (17). The majority of the experimental data were obtained with air jets having

Figure 6 - Experimental and Theoretical σ Values for Free Jets

temperatures approximately equal to that of the ambient air.

One exception is the data of Anderson and Johns, which were obtained with a solid propellant rocket exhausting into still air. For Mach numbers less than unity, the value of σ has been relatively well established as being $\sigma = 12.0$. For Mach numbers greater than unity, the data illustrate that σ increases considerably.

For the turbulent mixing between two compressible coaxial streams, analytical attempts (18) have resulted in relating the two-stream σ -value and the velocity ratio between the two streams. The equivalent single stream value is obtained from Korst's empirical relationship

$$\sigma_I = 12 + 2.758 M_{Ia} \quad (65)$$

where M_{Ia} is an equivalent single stream Mach number related to the jet stream Crocco number as shown in Fig. 7. Also shown in Fig. 7 is the ratio of the aforementioned σ -values as a function of the velocity ratio between the two streams; indicating that the two stream σ -value increases with increasing velocity ratio. This trend has also been established from experimental data (12).

Since experimental data for predicting two stream σ -values are limited, the above technique for evaluating σ has been adopted herein with the exception that the equivalent single stream σ_I value is obtained from Fig. 6 corresponding to the equivalent single stream Mach number. The corresponding value of Reichard's coefficient is then determined from Equation 64.

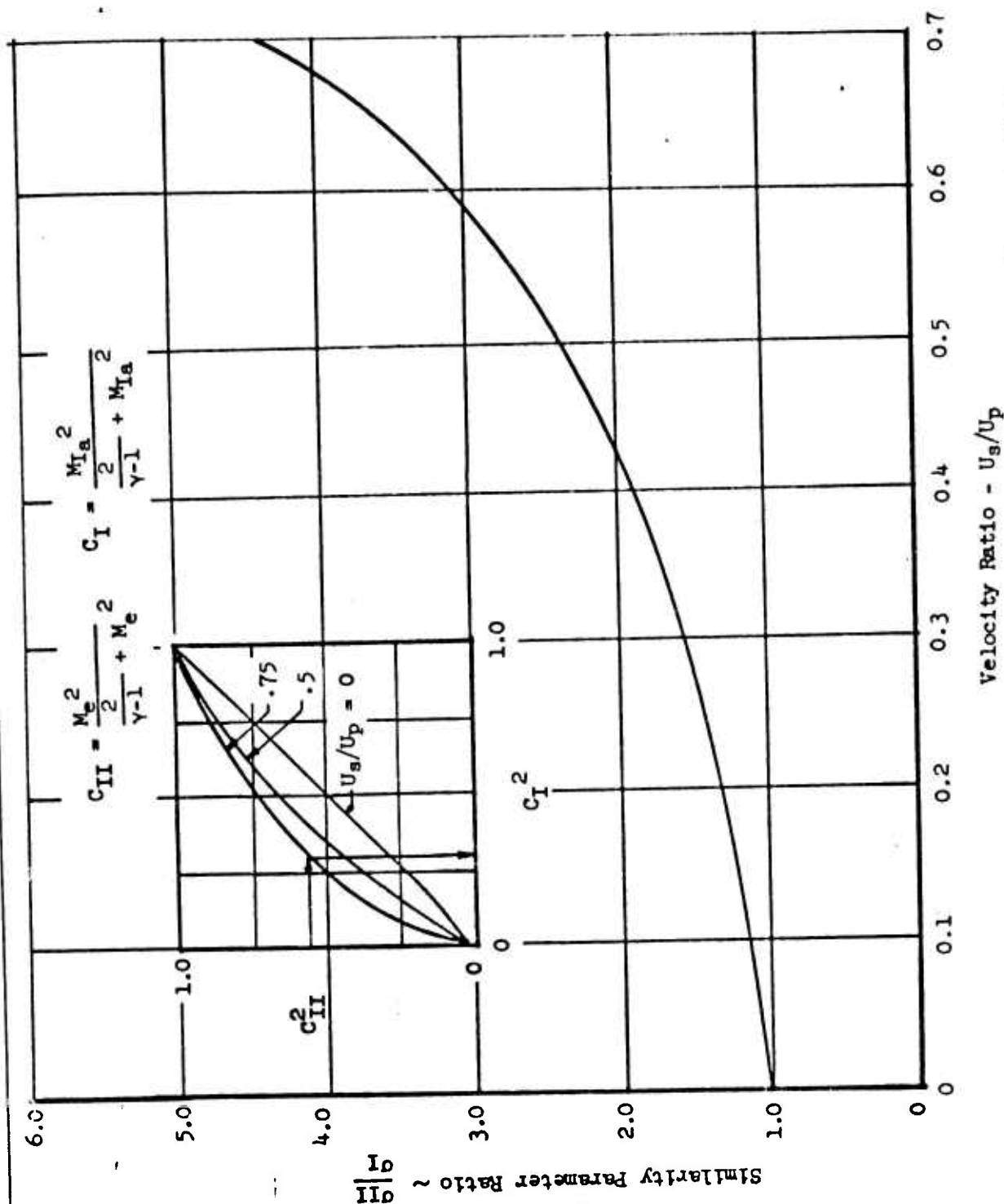


Figure 7 - Theoretical Effect of Velocity Ratio on Similarity Parameter Ratio

REV LTR

U3 4288-2000 REV. 6/64

BOEING

NO. D2-36251-1

SH. 48

It should be noted that in utilizing any of the above ϵ_v expressions, difficulties may arise resulting from the method employed in correlating experiment and theory. The accepted method of establishing the validity of a particular model is to employ a specific theoretical analysis and several trial values of the empirical constant in the ϵ_v expression, until agreement is obtained between the measured and calculated velocity, temperature, and concentration profiles. Unfortunately, the resulting ϵ_v expressions reflect any peculiarities of the particular theoretical analysis used in the correlation, and consequently should be used with reservation for other analyses.

3.4 Turbulent Lewis and Prandtl Numbers

The turbulent Lewis and Prandtl numbers are the turbulent counterparts of the corresponding numbers based on molecular properties. By definition

$$Le_i = \frac{\epsilon_{Di}}{\epsilon_H} \quad (66)$$

$$Pr = \frac{\epsilon_v}{\epsilon_H} \quad (67)$$

Another dimensionless ratio frequently employed is the Schmidt number defined by

$$Sc_i = \frac{Pr}{Le_i} = \frac{\epsilon_v}{\epsilon_{Di}} \quad (68)$$

where the subscript i infers that the various species may have different eddy mass diffusivities. In early mixing analyses it was often assumed, in analogy with the molecular ratios, that $Pr = Sc = Le = 1$; an assumption

which simplifies the governing equations considerably. However, recent experimental data indicate that these parameters may differ markedly from unity (14, 19). For example, from the experimental results of Forstall and Shapiro (19), it was found that the turbulent Pr and Sc numbers were equal and constant throughout the mixing region, with a value of approximately 0.7. From the experimental results of Zakkay, et al, (14) it was concluded that the turbulent Schmidt and Lewis numbers ranged from 0.3 to 2.3 and 0.4 to 1.0, respectively.

In the present analysis, values of the turbulent Prandtl and Lewis numbers may be employed other than unity, however they remain as constants independent of the x-y coordinates. For gaseous components the values of Pr and Sc most frequently cited in the literature vary between 0.5 and 1.2. It was shown in Ref. 20 that variations of Sc have negligible effects on the radial profiles, so that any value within the above range may be used with reasonable accuracy.

The Lewis and Prandtl numbers for the solid and/or liquid particles in gas-particle mixtures are at the present time unknown. The experimental data of Longwell and Weiss (21), however, indicate that the ratio of gas to liquid eddy mass diffusivities is approximately 2. In their studies liquid diesel fuel was injected into a turbulent air stream and concentrations measured at various axial locations. The results were compared to similar data obtained with naphtha as the injected fuel. As noted by the authors the naphtha evaporated readily and was assumed to be mainly in the gaseous phase. The results showed that the ratio of the gas to liquid ϵ_p varied from approximately 1.2 to 2.0 for gas stream velocities ranging from 200 to 500 fps, respectively.

In view of the lack of experimental data it is difficult to ascertain values that should be employed for the turbulent Lewis and Prandtl numbers in order to accurately predict flow profiles. However, until reliable experimental data are available, the values cited above may be employed with results within the accuracy required in most engineering problems.

REV LTR

U3 4288-2000 REV. 6/64

BOEING

NO.

D2-36251-1

SH.

51

4.0 CHEMICAL MODEL

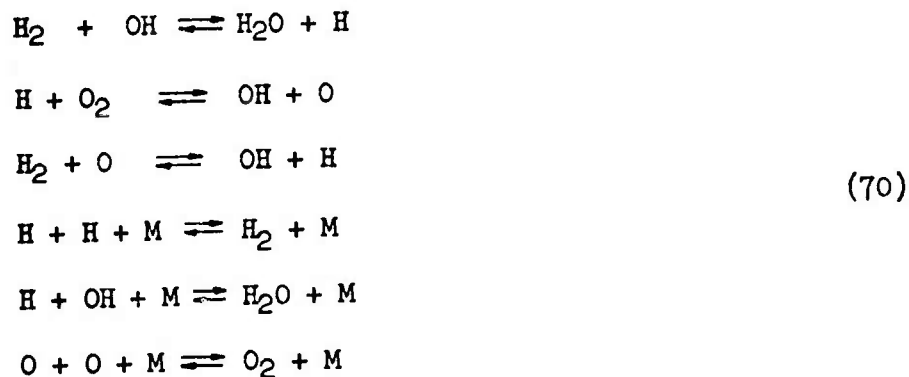
4.1 General

In considering the chemical aspects of problems involving the mixing between reactive components, two choices of chemical behavior are available; namely, equilibrium or nonequilibrium chemistry. Because of the high velocities encountered in rocket exhaust jets, it appears that in such problems nonequilibrium chemistry would prevail throughout most of the flow field. The most accurate analysis should, therefore, be based on nonequilibrium flow including the complicated chemical equations describing the reaction mechanism. However, in view of the complexity of such an analysis and the resulting lengthy calculations, it is advantageous to use equilibrium chemistry whenever that choice leads to accurate results.

In the discussion that follows the equations for both equilibrium and nonequilibrium chemistry are developed for the hydrogen-oxygen system and the results compared. From the comparison of temperature profiles, it was ascertained that the assumption of equilibrium chemistry was valid for hydrogen-oxygen systems. This assumption was therefore employed for such systems and extended to other systems as discussed below.

4.2 Nonequilibrium Chemistry

For nonequilibrium chemistry the net rate of production of the species w_k , is determined using the methods of theoretical reaction kinetics. For example, consider the chemical reaction between hydrogen and oxygen forming water vapor. It is well known that the reaction mechanism between these molecules actually consists of several chain-type reactions including the following:



where M is a third body usually taken to represent all the molecules in the mixture.

In order to determine the net rates of production of the various species entering the above reactions, it is necessary to apply the reaction kinetics equations to each of the reaction equations. In order to simplify the present analysis, however, it is assumed that the above complex chain reactions for the $\text{H}_2\text{-O}_2$ system may be replaced by the following one-step reaction equation



where k_f is an "overall" reaction rate constant.

Following Penner (22) it can be shown that the net rate of production of H_2O , in accordance with reaction equation 71, may be expressed as

$$\dot{w}_1 = 2W_1 k_f' \rho^3 \frac{C_2}{W_2} \left(\frac{C_2}{W_2} \right)^2 \left[1 - \frac{W_3}{K_p' RT \rho C_3} \left(\frac{C_1}{W_1} \frac{W_2}{C_2} \right)^2 \right] \tag{72}$$

where k_f' is the forward reaction rate constant, K_p' is the equilibrium constant, and W is the molecular weight. The subscripts 1, 2, 3 correspond to the

species H_2O , H_2 and O_2 , respectively.

From further application of the reaction rate equations it may be shown that the net rate of production of H_2 is related to that of H_2O . Thus

$$\dot{w}_2 = -\dot{w}_1 \frac{w_2}{w_1} \quad (73)$$

From conservation of mass the net rate of production of O_2 becomes

$$\dot{w}_3 = -(\dot{w}_1 + \dot{w}_2) \quad (74)$$

The expression for the reaction rate constant k_f is assumed to be of the form

$$k_f = BT^{\gamma} \exp\left(-\frac{E}{RT}\right) \quad (75)$$

where B is the frequency factor, E is the activation energy, and γ is a constant.

Equations (72) through (74) may be employed in solving the finite difference energy and species conservation equations provided the empirical constants in equation (75) may be established. The primary difficulty in all non-equilibrium problems is that of establishing the reaction mechanism and associated expressions for the reaction rate constants. For the H_2 - O_2 system the complex chain reaction equations and rate constants are relatively well established. It is possible therefore, to establish an expression for an "overall" reaction rate constant in the following manner. Utilizing the chain reactions and the associated reaction rate constants, the temperature distribution throughout a stream tube may be calculated for a range of initial

conditions of the flow parameters employing the one-dimensional analysis of Ref. 23. Then, using the same analytical technique and the one-step chemical reaction a value of the overall reaction rate constant may be determined which best reproduces the previously determined temperature distribution.

The above technique was employed for a range of initial conditions corresponding to those anticipated for the problem being considered herein. Typical results are shown in Fig. 8 wherein are presented the calculated temperature distributions resulting from the use of chain reactions (the solid line) and the one-step reaction using various expressions for the overall reaction rate constant (the broken lines). The reaction equations employed in calculating the solid line were those presented in equation 70 herein. The value of the overall reaction rate constant was varied by altering the frequency factor; the activation energy, obtained from Ref. 3, was assumed constant at 1.6×10^4 cal/gm-mole. The average value of the frequency factor which best correlated the results obtained from the complex reactions was $B = 10^{17}$. Thus, the expression employed for the overall reaction rate constant for the H_2O_2 reaction was determined to be

$$k_f = 10^{17} T^{0.5} \exp \left(\frac{-1.45 \times 10^4}{T} \right) \quad (76)$$

The w_1 terms appearing in the finite difference equations are then determined from equations 72-74.

4.3 Equilibrium Chemistry

Employing the assumption of equilibrium chemistry the net rate of production

of each species may be determined from the equation

$$\dot{w}_i = \rho u (c_i - \bar{c}_i) \quad (77)$$

where \bar{c}_K and c_K are the concentrations of species K immediately before and after the chemical reaction, respectively. The problem, therefore, reduces to that of calculating the equilibrium composition c_K , for initial concentrations \bar{c}_K at a given pressure. In the discussion that follows the equilibrium equations are developed not only for the hydrogen-oxygen system, but in addition, for systems comprising the following species: H_2O , H_2 , O_2 , CO_2 , CO , N_2 , HCl , Al , and Al_2O_3 . The chemical reaction equations are assumed to be as follows:



Consider now the elemental control volume shown in Fig. 2. As the fluid traverses the distance Δx , the concentrations change as a result of the mixing process. If it is assumed that the fluid entering the volume is in chemical equilibrium, then at $x + \Delta x$ the fluid is not in equilibrium. Since the assumption of equilibrium flow implies that the reactions occur at an infinite rate, the composition at $x + \Delta x$ may be determined as follows. From an atom balance the following relations may be obtained which are valid for both reactants and products:

$$\tilde{c}_2 = \bar{c}_1 + \bar{c}_1 \frac{w_2}{w_1} \quad (81)$$

$$\tilde{C}_3 = \bar{C}_3 + \bar{C}_1 \frac{W_3}{2W_1} + \bar{C}_4 \frac{W_3}{W_4} + \bar{C}_5 \frac{W_3}{2W_5} + \frac{3}{2} \frac{W_3}{W_{10}} \bar{C}_{10} \quad (82)$$

$$\tilde{C}_4 + \tilde{C}_5 \frac{W_4}{W_5} = \bar{C}_4 + \bar{C}_5 \frac{W_4}{W_5} \quad (83)$$

$$\tilde{C}_6 = \bar{C}_6 \quad (84)$$

$$\tilde{C}_7 = \bar{C}_7 \quad (85)$$

$$\tilde{C}_8 = \bar{C}_8 \quad (86)$$

$$\tilde{C}_9 = \bar{C}_9 + \frac{2W_9}{W_{10}} \bar{C}_{10} \quad (87)$$

where: \tilde{C}_i is a pseudo-mass fraction defined by the above equations and the subscript $i = 1, 2, \dots, 10$ refers to the species $H_2O, H_2, O_2, CO_2, CO, N_2, HCl$, any inert component, Al, Al_2O_3 , respectively. Equations 81-87 are also valid for the products - the \bar{C}_i on the right side replaced by c_i .

The equilibrium constants for the reaction equations 78-80, expressed in mass fractions, are

$$K_P^1 = \frac{W_3}{C_3 P W_M} \left(\frac{C_1 W_2}{W_1 C_2} \right)^2 \quad (88)$$

$$K_P^2 = \frac{W_3}{C_3 P W_M} \left(\frac{C_4 W_5}{W_4 C_5} \right)^2 \quad (89)$$

$$K_P^3 = \left(\frac{W_9}{C_9} \right)^2 \left(\frac{W_3}{C_3} \right)^{3/2} (P W_M)^{-7/2} \quad (90)$$

where the fugacity of Al_2O_3 is assumed to be unity (24), and W_M is the average molecular weight of the gas mixture. Values of the equilibrium constants are obtained from tables of K_p at various temperatures (25). Equations 81-90 are seen to form a system of ten equations for the ten unknown concentrations c_i . The solution for the concentrations comprises an iteration technique discussed in Ref. 6. The net rate of production terms are then calculated from Equation 77.

4.4 Comparison of Equilibrium and Nonequilibrium Chemistry

In order to illustrate the difference between equilibrium and nonequilibrium chemistry, solutions were obtained for a typical problem employing both chemical models for the hydrogen-oxygen system. Shown in Fig. 9 is the temperature profile throughout the mixed flow field at an axial distance of four feet downstream from the exhaust nozzle of a LO_2/LH_2 rocket motor. The conditions of the rocket exhaust and secondary air stream at the exit plane of the nozzle are noted in the figure. As can be seen, the difference between the two cases is not great, thus demonstrating that the selection of equilibrium chemistry provides sufficiently accurate results for the given conditions. As a point of interest, it is worthwhile to note that the machine computation times for equilibrium and nonequilibrium flow were approximately 2 and 110 minutes, respectively. In view of the long computation times required for nonequilibrium flow, and since the assumption of equilibrium flow yields reasonably accurate results, the latter chemical model was assumed for the problem under study herein.

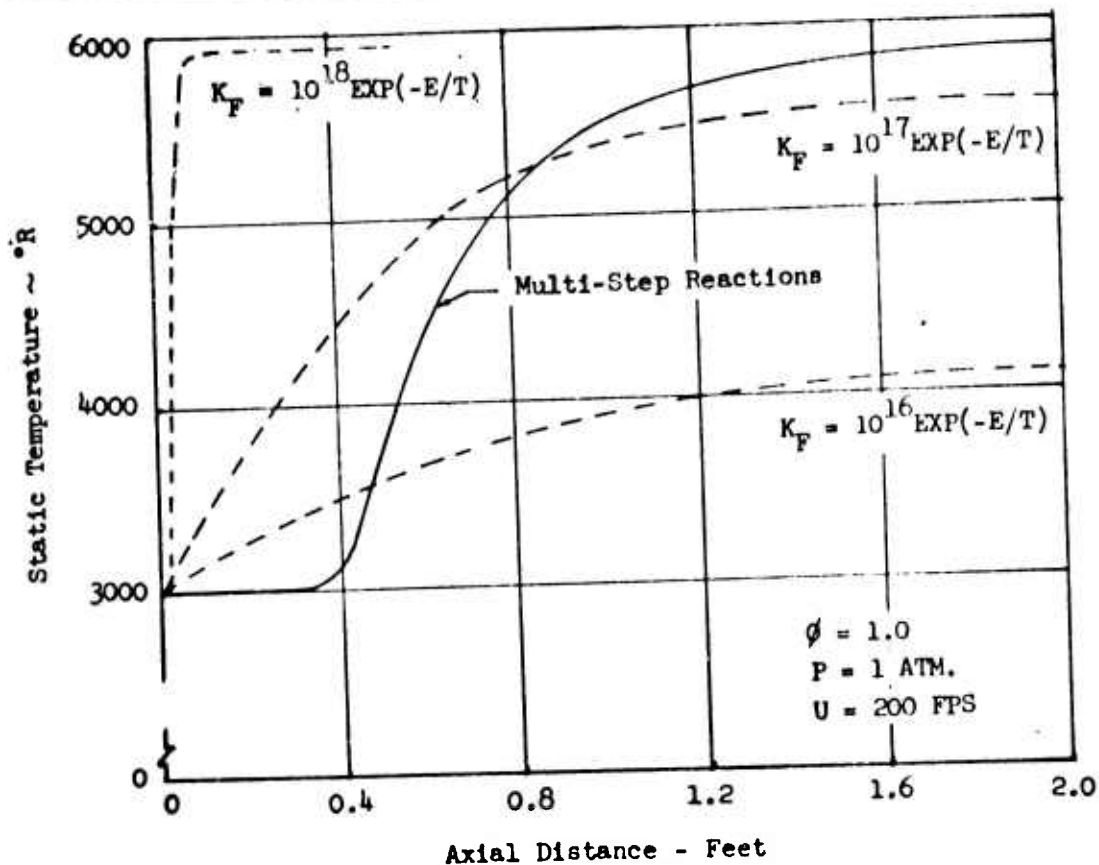


Figure 8 - Effect of Reaction Mechanism and Rate Constants on Theoretical Static Temperature Distribution

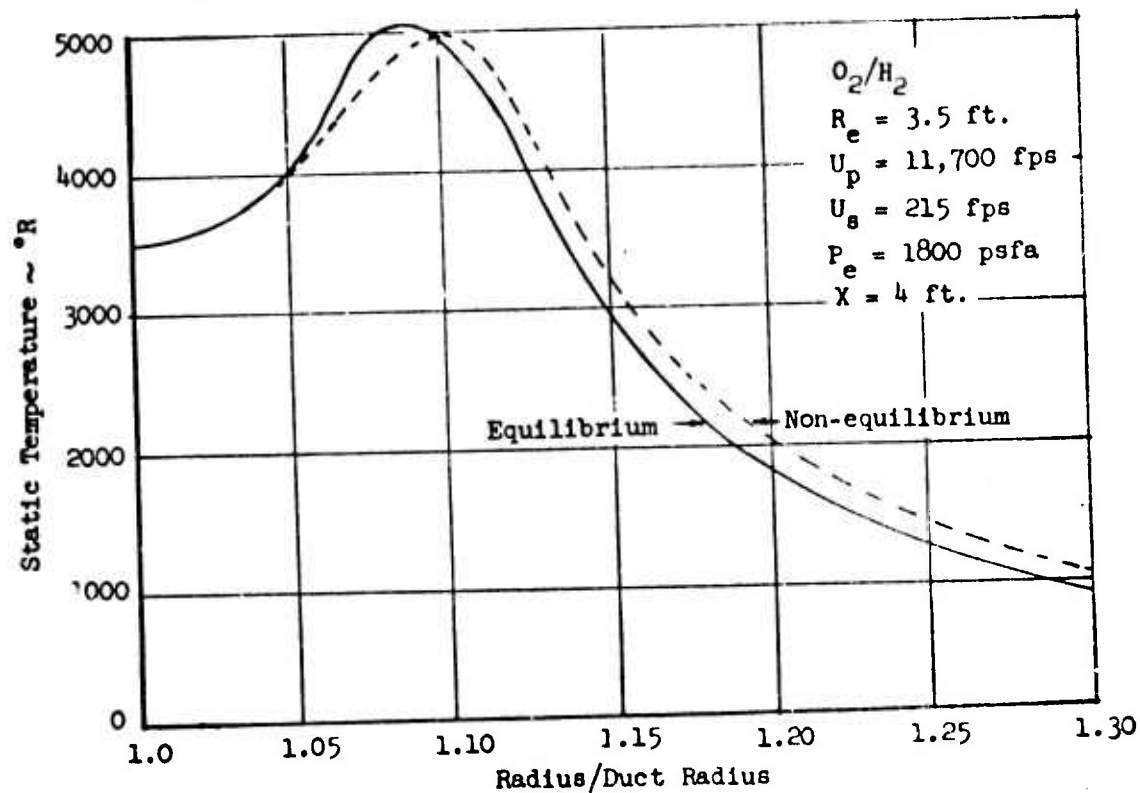


Figure 9 - Comparison of Theoretical Temperature Profiles for Non-Equilibrium and Equilibrium Flow

5.0 COMPUTER PROGRAM

5.1 Capability

Although the computer program described herein was developed specifically for the air-augmentation problem it may, however, be employed for other related problems. For example, the program may be utilized in calculating the mixing region for free jet problems, where the ambient atmosphere is either moving or at rest. In addition, calculations may be performed for problems involving the mixing and combustion of fuel injected into an air stream, with application to ramjet combustor studies. Problems involving chemically reactive species may be solved employing either equilibrium or frozen chemistry. It is noted that the program is limited to the calculation of turbulent flow fields - laminar flow problems being excluded.

In the following sections will be described the input options available to the user, the important features of the program, and the procedure to be followed in establishing the input data for typical problems. For convenience in the discussion that follows a list of the pertinent computer program notation is presented in Appendix 7.1.

5.2 Pressure or Duct Profile Option

In air-augmentation problems it is generally desirable to specify the duct geometry; whereas, in free jet problems it is more convenient to specify the pressure distribution. In the computer program the option is available to specify either the duct profile in the form $r_D = f(x)$, or the pressure

distribution in the form $p = g(x)$. The program control for this option is termed INDUCT - having values of 0 or 1 for pressure distribution or duct geometry input, respectively. The input of $p = g(x)$, or $r_D = f(x)$, is in the form of a table. The x-values are termed XTAB and the p- or r_D - values termed PTAB in program notation. It should be noted that, from the viewpoint of machine time, it is advantageous to input the pressure distribution whenever possible.

If the pressure input option is chosen the duct radius is calculated from the initial specified duct radius, denoted RDZERO, and conservation of mass of the flow through the duct. Conversely, if the duct input option is chosen, the pressure at each front is calculated from the initial specified pressure PZERO, and conservation of mass. Problems involving free jets are treated by specifying RDZERO very large compared to the nozzle exit radius RE.

5.3 Reference Streamline

In performing the finite-difference calculations it is necessary that the coordinates of a reference streamline be specified, i.e., $\psi_{\text{ref}} = f(x, r)$. Thus the inverse transformation, given by Eqn. 40, may be performed. In the present program the reference ψ -line may be specified as coincident with either the rocket exhaust plume boundary or the centerline of the flow field as shown in Fig. 10. In the former case the coordinates of the plume slipline may be determined utilizing the method of characteristics program of Ref. 5. The slipline coordinates are input to the program in tabular form, denoted as XTAB and RTAB. The input value of RZERO1 is

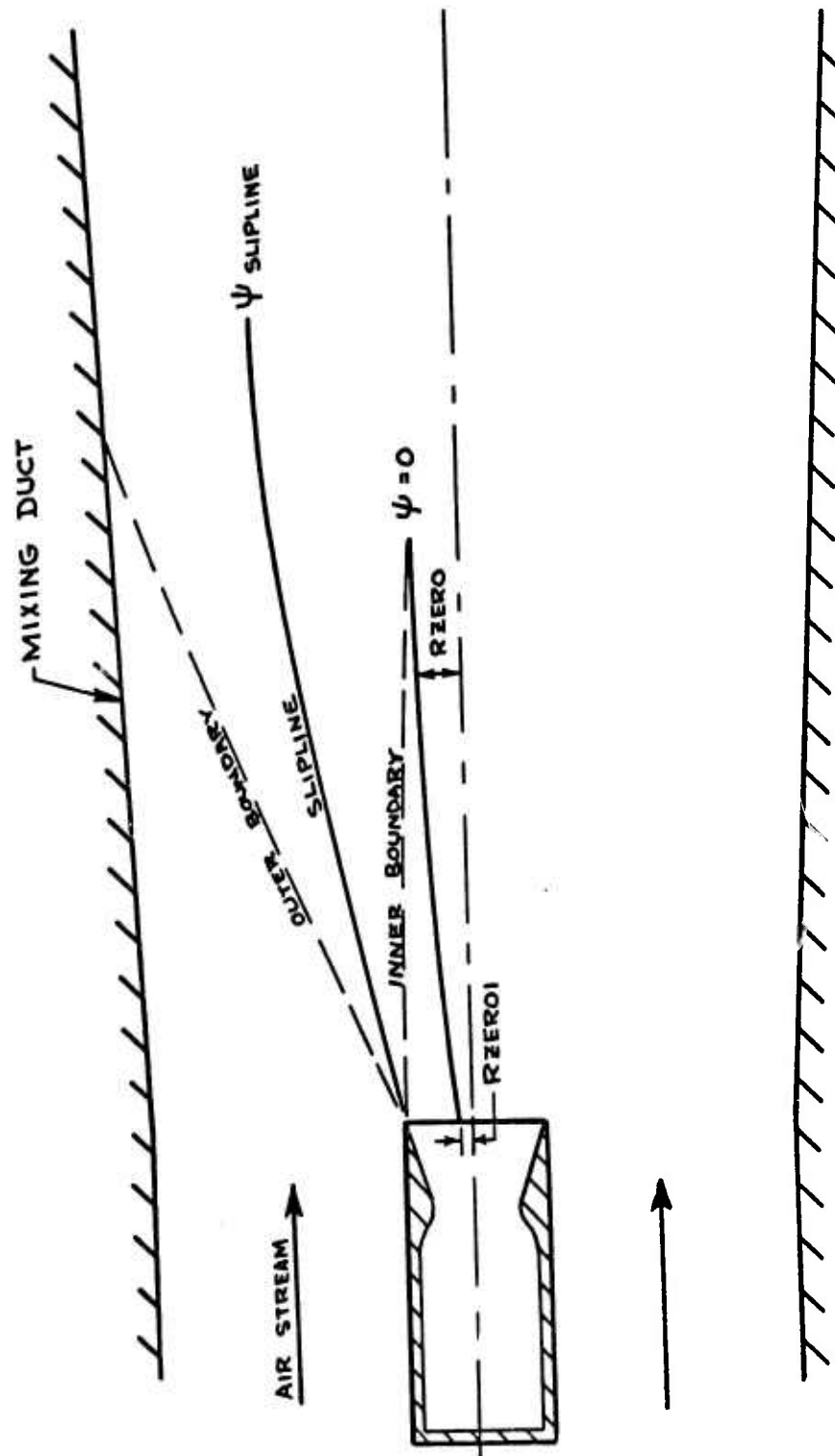


FIG. 10 Location of Reference Streamline Coincident with Plume Slipline

REV LTR

U3 4288-2000 REV. 6/64

BOEING

NO. D2-36251-1

SH.

62

accordingly specified as being slightly larger than zero, e.g., RZERO1 = 0.001 is usually adequate. One problem which is generally encountered using the slipline as ψ_{ref} is as follows. As the calculations proceed in the x-direction the mixing zone widens, until at a certain distance the mixing zone crosses the $\psi = 0$ streamline. Once this condition occurs the reference ψ -line is shifted to the jet centerline and the calculations continued, or if desired the program is stopped. The control parameter for this option is termed ISET - having values of 0 or 1 for stopping or continuing the program, respectively. For cases where it is desired to specify the centerline as the reference ψ -line the value of RZERO1 and RTAB are set equal to zero and the radius of the jet, respectively.

5.4 Mixing Zone Definition

The width of the mixing zone, as defined herein, is determined from the species concentrations. At a given "front", the calculation of all flow properties begins in the mixing region and proceeds inward and outward until the edges of the mixing region, as established from the following criteria, are determined. At each grid point the difference is calculated between the concentration of species i at the grid point in question and the outer boundary condition of C_i , denoted as C_{iNMAX} . This quantity is calculated for each of the species, $i = 1-10$, and the maximum difference determined. The maximum difference is then compared with an input quantity termed TESMAX. The outer edge of the mixing region is established when

$$(C_{iN} - C_{iNMAX})_{MAX} \leq TESMAX \quad (91)$$

Similarly, the inner edge of the mixing zone is established when

$$(C_{iN} - C_{iNMIN})_{MAX} \leq TESMIN \quad (92)$$

where C_{iNMIN} is the inner boundary condition of C_i , and TESMIN is an input quantity. The values of TESMIN and TESMAX should be sufficiently small so that the mixing zone boundaries determined from species concentrations are larger than the corresponding boundaries determined from velocity or temperature profiles. A value of 0.00001 for both TESMIN and TESMAX is sufficient for most problems - smaller values yield wider mixing zones.

5.5 Stability Criteria

The major difficulties in solving differential equations utilizing the finite-difference technique involve the problems of stability and convergence of the solution. A stable solution is defined as one where small errors, which are inherent in the calculations, do not increase in size. In unstable solutions the errors increase in size during the calculations and eventually become so large that the results are noticeably inaccurate. A solution is defined as convergent if by decreasing the grid size (Δx and $\Delta \psi$) the calculated velocity, temperature, and concentration profiles are not significantly altered. It has been established that if a solution is stable it is also convergent (7, 26).

Employing Karplus' theory of stability it was determined that for the present problem the species continuity equation dictated the most stringent stability criteria (5). Thus in order to insure a stable solution, and consequently a convergent solution, the stability criteria dictates

that

$$\Delta\psi \geq \left(2 \rho^2 u r^{1\delta} \epsilon_v \Delta x \frac{Le}{Pr} \right)^{1/2} \quad (93)$$

where

$$\epsilon_v = \epsilon_o \left(C + x/L \right)^n \quad (94)$$

In application, the right side of Eqn. 93 is evaluated at each grid point in the mixing region along a given front, and the maximum value determined. For stability the value of $\Delta\psi$ must be greater than or equal to the established maximum value. As can be seen the value of the right side of Eqn. 93 is related to the axial distance. Thus, it becomes evident that if constant values of $\Delta\psi$ and Δx are selected, special consideration must be given to the selection of their values so as to insure stability throughout the calculations.

In order to remove the above difficulty the value of Δx in the present program is assumed to be constant and the value of $\Delta\psi$ altered whenever required during the calculations. The most convenient method of changing $\Delta\psi$ is to merely double its value whenever Eqn. 93 indicates that a change is required. Thus in the present program the stability test is performed at each front, and if required, $\Delta\psi$ is doubled—all the calculations being performed within the computer.

As a result of doubling $\Delta\psi$ it is apparent that certain streamlines, which were calculated prior to the change of $\Delta\psi$, are omitted in subsequent calculations. For example, in the program the ψ -lines are numbered consecu-

tively with the reference ψ -line, which is retained throughout the calculations, assigned the number 1. Then as a result of doubling $\Delta\psi$ the even numbered ψ -lines are omitted in the remaining calculations. After the doubling process the ψ -lines are again numbered consecutively. Thus the ψ -lines numbered $N = 3, 5, 7, \dots$ before the doubling process, become $N = 2, 3, 4, \dots$ in subsequent fronts. The general equation for tracing an odd numbered ψ -line is

$$n' = \frac{n+1}{2}$$

where n and n' are the numbered ψ -lines immediately before and after doubling $\Delta\psi$, respectively.

5.6 Determination of Initial Grid Size

The initial grid size, Δx and $\Delta\psi$, may be established from the stability criteria and the maximum possible number of grid points along the initial front ($x = 0$). Because of computer storage limitations the maximum number of radial grid points is limited to 196. In order to trace the ψ -line originating at the outer periphery of the nozzle it is advantageous to set $KSLIP = 129$. Thus, for given initial conditions of the jet, the initial value of $\Delta\psi$ may be calculated from the expression

$$\Delta\psi_{INITIAL} = \frac{(\rho U)_{NMIN} (RE)^{\delta+1}}{(\delta+1) KSLIP} \quad (95)$$

Once $\Delta\psi$ is established the value of Δx may then be determined from the

stability criteria. Thus from Eqn. 93

$$\Delta x \leq \frac{\Delta \psi^2 Pr}{2 \rho^2 u r^{2.5} \epsilon \nu Le} \quad (96)$$

where, initially, the value of x in Eqn. 94 may be taken as Δx , and the product $\rho^2 u r^{2.5}$ is evaluated at the radius of the jet RE. This value of Δx is sufficient to insure stability at the first front.

However, for subsequent fronts it is generally not sufficient, so that a value somewhat smaller than that given above should be employed. From a consideration of machine time the value of Δx generally should be such that

$$\frac{x_{MAX}}{\Delta x} < 3000$$

A smaller value of Δx than that given above may be employed if required; however, it should be noted that machine time may be increased significantly.

5.7 Input Procedure

In order to describe the input procedure it is convenient to refer to Fig. 11 wherein is presented the standard input form for the computer program. The method of selecting the values for each parameter on the input form is discussed below.

NMIN (Two Options)

The two options available for NMIN are 0 or the number assigned to the ψ -line corresponding to the inner edge of the mixing region. The

purpose of the latter option is to permit the input of variable flow properties at the exit plane of the nozzle ($x = 0$), or at any x -location if desired.

NMAX (Two Options)

The two options are $NMAX = 196$ or the number assigned to the ψ -line corresponding to the outer edge of the mixing region. The purpose of the latter option is discussed above.

KSLIP

KSLIP is the number assigned to the ψ -line originating at the outer periphery of the nozzle ($r = RE$). In most problems it is desirable to set $KSLIP = 129$ for purposes of tracing that ψ -line; i.e., not losing the ψ -line when $\Delta\psi$ is doubled. Other values which may be employed for KSLIP are 65, 33, 17, 9, or 5.

NS

The parameter NS is employed to permit a curve fit of the flow properties between the rocket exhaust and secondary streams at $x = 0$, thereby removing the discontinuity of properties at $r = RE$. The value of NS may be $NS = 0$ or any small integer, e.g., $NS = 3$ is generally employed.

NCHEM (Two Options)

The two options are $NCHEM = 0$ or 1 corresponding to equilibrium or frozen chemistry, respectively.

NCP8

The value of NCP8 is the number of temperature values in the table of

TCP8 vs C_{p8} for the general species.

NXTAB

The value of NXTAB is the number of x-values in the table of XTAB vs. PTAB.

INDUCT (Two Options)

The two options are INDUCT = 0 or 1 corresponding to the selection of pressure distribution or duct geometry input, respectively.

ISSET (Two Options)

The two options are ISSET = 0 or 1 corresponding to the following conditions:

ISSET = 0; if RZERO1 \neq 0 the program will stop when the inner edge of the mixing zone reaches the $N=1$ ψ -line.

ISSET = 1; the program will not stop as above, but will shift the reference ψ -line to the centerline and continue the calculations.

DX

The axial step size may be determined using the method outlined in Section 5.5.

XMAX

The maximum axial distance over which the calculations are desired is to be expressed in feet.

XPRINT

The value of XPRINT should be selected so that the ratio XPRINT/DX is an integer.

R ZERO1

The initial r -value of the reference ψ -line may be selected as 0 or any positive value depending on whether the reference ψ -line is desired to be located on the jet centerline or the plume slipline, respectively (see Section 5.3).

RE

The value of the nozzle radius is to be expressed in feet.

RDZERO

The value of the initial duct radius is to be expressed in feet.

PZERO

The initial pressure in the secondary stream is to be expressed in PSFA.

DPDX0

The initial value of the pressure gradient may be input if known. Otherwise, the value 0 may be employed.

DELTA (Two Options)

The two options are 0 to 1 corresponding to a two-dimensional or axisymmetric flow model, respectively.

TESMAX and TESMIN

The values employed for TESMAX and TESMIN determine the width of the mixing zone defined by species concentration - smaller values yielding wider mixing zones. A value of 0.00001 is generally satisfactory for both parameters (see Section 5.4).

A, FL, EPSON, and FCON

A complete discussion of these parameters is presented in Section 3. Values generally employed for the above parameters are

$$A = FL = 1$$

$$FCON = 0$$

The value of EPSON must be determined for each problem from empirical expressions as discussed in Section 3.3.

PRT, FLETG, and FLETP

The turbulent Prandtl and Lewis numbers of the gas may be specified as 0.8 and 1.2, respectively. A particle Lewis number of 0.6 may be employed until data become available for more accurately establishing its value.

XTAB, RTAB, and PTAB

The data for the plume slipline coordinates and the pressure distribution (or duct radius) is input in tabular form. For cases where the slipline geometry is unknown a value of RTAB equal to the nozzle exit radius RE , should be employed for all XTAB values. It should be noted that the input of pressure or duct radius at various axial distance XTAB, must be compatible with the input of the control parameter INDUCT.

GENN and W_g

If the user desires to incorporate an additional chemical species, other than any of those specified in the program, the chemical symbol of the

component is input for GENN and the corresponding molecular weight for W_g .

T_{CP8} and C_{P8}

If an additional chemical species is included the specific heat of the component as a function of temperature is input in tabular form. It should be noted that this table, as well as GENN and W_g , are input only if an additional species is desired. Otherwise, these parameters may be omitted.

G_1 , T , and U (Two Options)

The two options available to the user correspond to whether or not it is desired to input variable properties as the initial conditions. If the variable property option is selected NMIN must be different than zero. If the constant property option is selected the value of NMIN must be zero.

The variable properties are input as a function of N . The constant properties are input for the rocket exhaust (C_{i_1} , T_1 , u_1) and the secondary stream ($C_{i_{NMAX}}$, T_{NMAX} , u_{NMAX}).

6.0 REFERENCES

1. Abramovich, G. N., The Theory of Turbulent Jets, The M.I.T. Press, Cambridge, Mass., 1963.
2. Libby, P. A., "A Theoretical Analysis of the Turbulent Mixing of Reactive Gases with Application to the Supersonic Combustion of Hydrogen", General Applied Science Laboratories, Technical Report No. 242, June 1961.
3. Vasiliu, J., "Turbulent Mixing of a Rocket Exhaust Jet with a Supersonic Stream Including Chemical Reactions", Journal of the Aerospace Sciences, Jan. 1962.
4. Mikhail, S., "Mixing of Coaxial Streams Inside a Closed Conduit", Journal of Mechanical Engineering Science, Vol. 2, No. 1, 1960.
5. Savage, L. E., "Air-Augmented Power Plant Design Program (Plumed Mix-Master)", The Boeing Company Report AS1927, June 1964.
6. Cate, E., "Air-Augmented Rocket Mixing Program", The Boeing Company Report AS1872, (to be released).
7. Wu, J. C., "On the Finite-Difference Solution of Laminar Boundary Layer Problems", Proceedings of the 1961 Heat Transfer and Fluid Mechanics Institute.
8. Van Driest, E. R., "Turbulent Boundary Layer in Compressible Fluids", Journal of Aeronautical Sciences, March 1951.

6.0 REFERENCES (cont'd)

9. Rohsenow, W. M. and Choi, H., Heat, Mass, and Momentum Transfer, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1961.
10. Schlichting, H., Boundary Layer Theory, 4th Edition, McGraw-Hill Book Company, Inc., 1960.
11. Taylor, G. I., "Statistical Theory of Turbulence", Proc. Roy. Society, A151, 1935.
12. Pai, S. I., Fluid Dynamics of Jets, D. vonNostrand Company, Inc., 1954.
13. Ting, L. and Libby, P. A., "Remarks on the Eddy Viscosity in Compressible Mixing Flows", Journal of Aerospace Sciences, Oct. 1960.
14. Zakkay, V., Krause, E., and Woo, S. D., "Turbulent Transport Properties for Axisymmetric Heterogeneous Mixing", Polytechnic Institute of Brooklyn Report ARL 64-103, June 1964.
15. Maydew, R. C. and Reed, J. F., "Turbulent Mixing of Axisymmetric Compressible Jets (in the Half-Jet Region) with Quiescent Air", Sandia Corporation Report SC-4764 (RR), March 1963.
16. Channapragada, R. S., "A Compressible Jet Spread Parameter for Mixing Zone Analyses", AIAA Journal, September 1963.
17. Korst, H. H.; Chow, W. L.; and Zumwalt, G. W.; "Research on Transonic

6.0 REFERENCES (cont'd)

- and Supersonic Flow of a Real Fluid at Abrupt Increases in Cross Section - Final Report", Engineering Experimental Station, Univ. of Ill., M.E., Tech. Report 392-5, December 1959.
18. Miles, J. B., "Stanton Number for Separated Turbulent Flow Past Relatively Deep Cavities", Ph.D. Thesis, Univ. of Ill., 1963.
 19. Forstall, W., Jr., and Shapiro, A. H., "Momentum and Mass Transfer in Coaxial Jets", J. Appl. Mech., 17, December 1950.
 20. Alpinieri, L. J., "An Experimental Investigation of the Turbulent Mixing of Non-Homogeneous Coaxial Jets", Polytechnic Institute of Brooklyn, PIBAL Report No. 789, August 1963.
 21. Longwell, J. P. and Weiss, M. A., "Mixing and Distribution of Liquids in High-Velocity Air Streams", Industrial and Engineering Chemistry, Vol. 45, No. 3, 1953.
 22. Penner, S. S., Chemistry Problems in Jet Propulsion, Pergamon Press, New York, 1951.
 23. Emmons, D. L., "A One-Dimensional Analysis of a Flow System in Chemical Non-Equilibrium with Application to Combustor and Exhaust Nozzle Flow Problems", The Boeing Company Document D2-35256, January 1964.
 24. Huff, V. N., Gorden, S., and Morrell, V. E., "General Method and Thermodynamic Tables for Computation of Equilibrium Composition and

6.0 REFERENCES (cont'd)

Temperature of Chemical Reactions", NACA Report 1037, 1951.

25. JANAF Thermochemical Tables

26. Karplus, W. J., "An Electric Circuit Theory Approach to Finite-Difference Stability", Trans. AIEE, 77, 1958.

27. Lamar, F., "Thermochemical Expansion and Combustion of a General Gas Mixture (TEXACO)", The Boeing Company Report AS1842, June 1964.

28. Fitch, R. E. and Emmons, D. L., "Investigation of Vehicle-Integrated Rocket Powerplants with Air Augmentation", Document No. D2-23217-1 (Confidential), The Boeing Company, June 1964

7.0 APPENDICES

7.1 Computer Program Notation

A	program notation for the exponent n, in Eqn. 94
C _{1NMAX}	outer boundary condition for the concentration of species i
C _{1NMIN}	inner boundary condition for the concentration of species i
CP8	specific heat of the general species (i = 8)
DELTA	$\delta = 0$ for two-dimensional model $\delta = 1$ for axisymmetric model
DPDXO	initial pressure gradient dp/dx
DX	axial step size Δx
EPSON	Reichardt's constant ϵ_0 , in Eqn. 94
FCON	constant C, in Eqn. 94
FL	reference length L, in Eqn. 94
FLETG	turbulent Lewis number of the gas
FLETP	turbulent Lewis number of the particles
GENN	molecular formula for the general species (i = 8)
INDUCT	INDUCT = 0 for pressure distribution input; INDUCT = 1 for duct profile input
ISSET	ISSET = 0 stops program when NMIN = 1 and $r_0 \neq 0$ ISSET = 1 program continues after shifting r_0 to the centerline
KSLIP	number assigned to the ψ -line originating at the lip of the nozzle ($r = r_0$)
NCHEM	NCHEM = 0 corresponds to equilibrium flow; NCHEM = 1 corresponds to frozen flow

7.1 Computer Program Notation (cont'd)

NCP8	number of temperature values in the table of CP8 vs T
NMAX	maximum number of ψ -lines desired (NMAX = 196)
NMIN	NMIN = 0 for constant property input; NMIN = 0 for variable property input
NS	number of ψ -lines within which the flow properties between the inner and outer streams are curve-fitted
NXTAE	number of values in the table of X vs PTAB
PRT	turbulent Prandtl number
PTAB	value of r_D (or P) in XTAB table
PZERO	initial static pressure in the outer stream
RDZERO	initial radius of the duct
RE	radius of the nozzle
RTAB	radius of the slipline in the XTAB table
RZEROL	initial radius of the reference ψ -line
T	static temperature
T_{CP8}	temperature values in the table of T vs CP8
TESMAX	parameter used in determining NMAX
TESMIN	parameter used in determining NMIN
V	axial velocity
W_8	molecular weight of the general species ($i = 8$)
XMAX	maximum axial distance over which the calculations are performed
XPRINT	axial increments for print-out of data

7.1 Computer Program Notation (cont'd)

XTAB values of x in the XTAB table

XZERO initial x-value

7.2 Comparison of Theory and Experiment

Theoretical results obtained from the computer program are compared herein with available experimental data. In establishing the correlations the model employed for the eddy viscosity, as described in Section 4, was employed. In certain cases the turbulent transport coefficients were varied about their predicted values in order to obtain better correlations. In such instances the variation from the predicted values is discussed. It is noted that analytical results were compared with data obtained from air-augmentation experiments; however, due to the classified nature of the experimental data the correlation is not presented herein (see Ref. 28).

In the experiments conducted by Maydew and Reed (15) radial static and total pressure surveys were made at various axial locations downstream of an air jet exhausting into a quiescent atmosphere. Shown in Figure 12 are the theoretically and experimentally determined velocity profiles at 11.5 inches downstream of an air jet having a Mach number of 1.96. The parameter y' is the radial distance measured from the point where the velocity has decayed to one-half its value at the inner edge of the mixing zone, σ is the similarity parameter, X_e is the effective length of the mixing zone, and V_1 is the velocity at the inner edge of the mixing zone. The solid curve represents the theoretical results obtained from the present analysis employing a value of Reichardt's coefficients corresponding to the value of σ determined experimentally by Maydew and Reed. As can be seen the predicted velocities agree quite well with the experimental values over the entire mixing region. Similar agreement was obtained between the experimental and theoretical velocity profiles at axial locations nearer the nozzle exit.

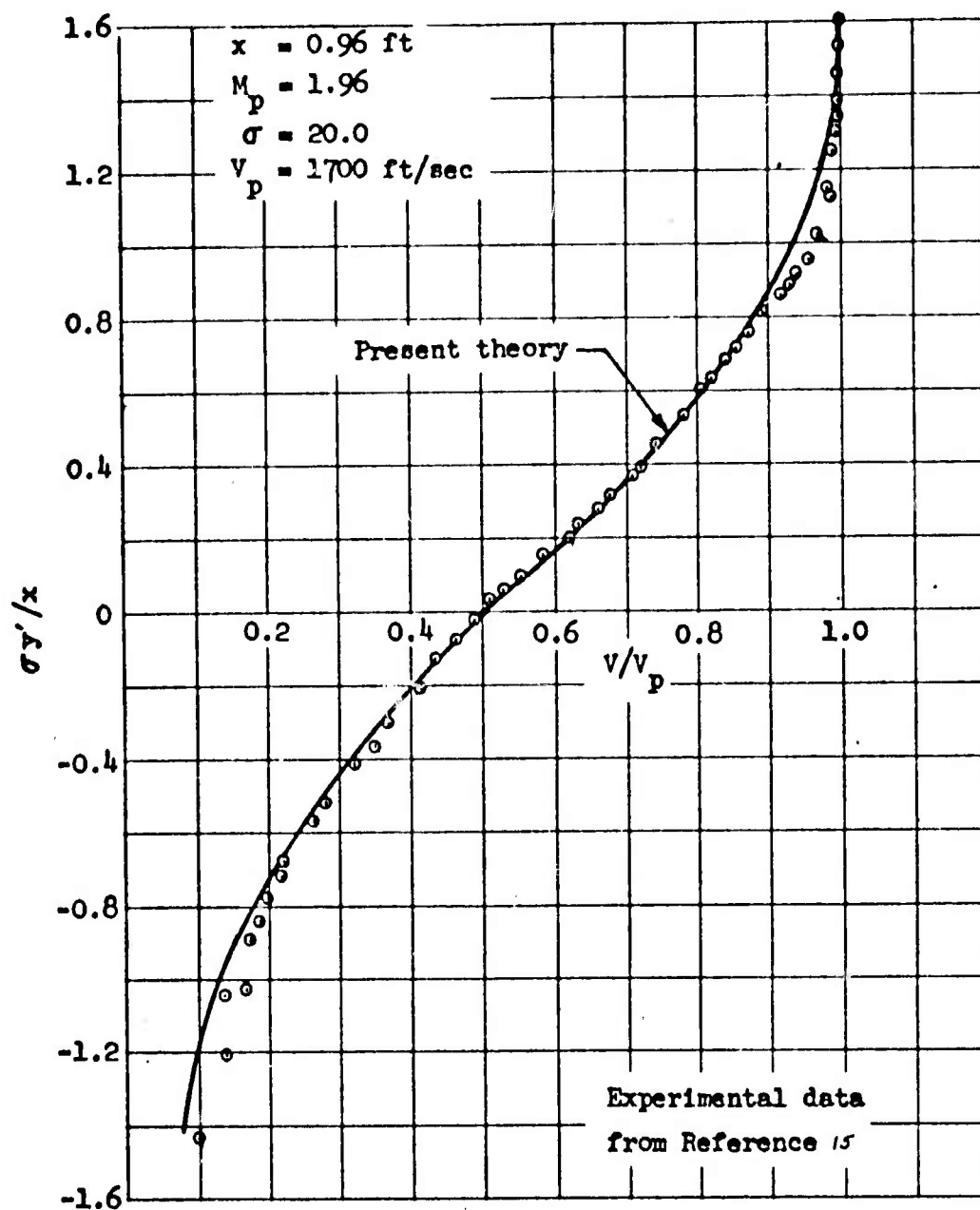


Figure 12 - Comparison of theory and experiment for free jet velocity profiles

In the experiments of Forstall and Shapiro (19) the mixing region between a circular jet and an annular coaxial stream was investigated. By using 10 percent by volume of helium as a tracer in the jet, the profiles of concentration and velocity in the mixing region were determined. Shown in Figure 13 is a comparison of the theoretical and experimental velocity and concentration profiles at an axial distance of $X/D = 8$. As can be seen reasonable agreement exists between the theoretical and experimental profiles. As noted in Ref. 19 the Prandtl and Schmidt numbers were both determined to be approximately 0.7, which corresponds to a Lewis number of unity. Theoretical results for two values of the Lewis number are shown for the concentration profiles in Figure 13. Small variations of the Lewis number are seen to effect little change in the profiles.

The experimental investigation of Zakkay, et al., was conducted to determine the turbulent mixing of coaxial jets for both subsonic and supersonic flow. In establishing a correlation, the data obtained with argon as the central stream was employed. The exit Mach number of the argon jet was approximately unity. The Mach number of the annular air stream was 1.6. Shown in Figure 14 is the comparison of the theoretical and measured velocity and concentration profiles at various axial locations along the jet centerline. In establishing the correlation the value employed for Reichardt's coefficient was $\epsilon_0 = 0.5 \text{ ft}^2/\text{sec}$ from $X = 0 - 0.42 \text{ ft}$, and $\epsilon_0 = 1.0 \text{ ft}^2/\text{sec}$ from $X = 0.42 - 0.75$. As can be seen good agreement exists between the theoretical and measured concentration profiles at each axial location. The velocity profile at $X = 0.75 \text{ ft}$ indicates that theory predicts slightly higher velocity ratios than those measured experimentally, which possibly may be attributed to an incorrect form for the eddy viscosity expression.

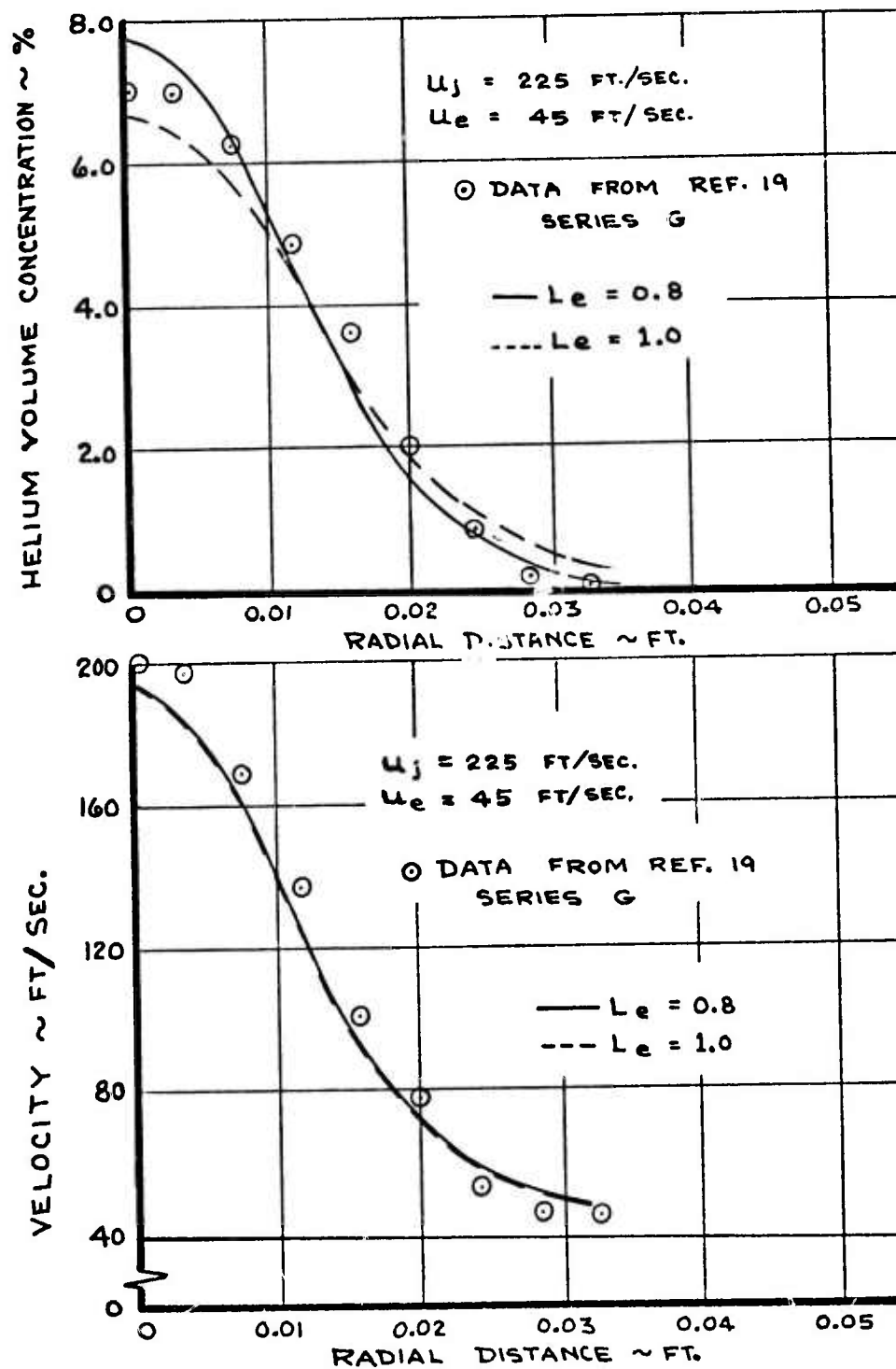


Fig. 13 COMPARISON OF THEORY AND EXPERIMENT

However, judging from the accuracy of the predicted concentration profiles and the fair agreement of the velocity profiles, it is concluded that the present eddy viscosity model is adequate for most engineering problems.

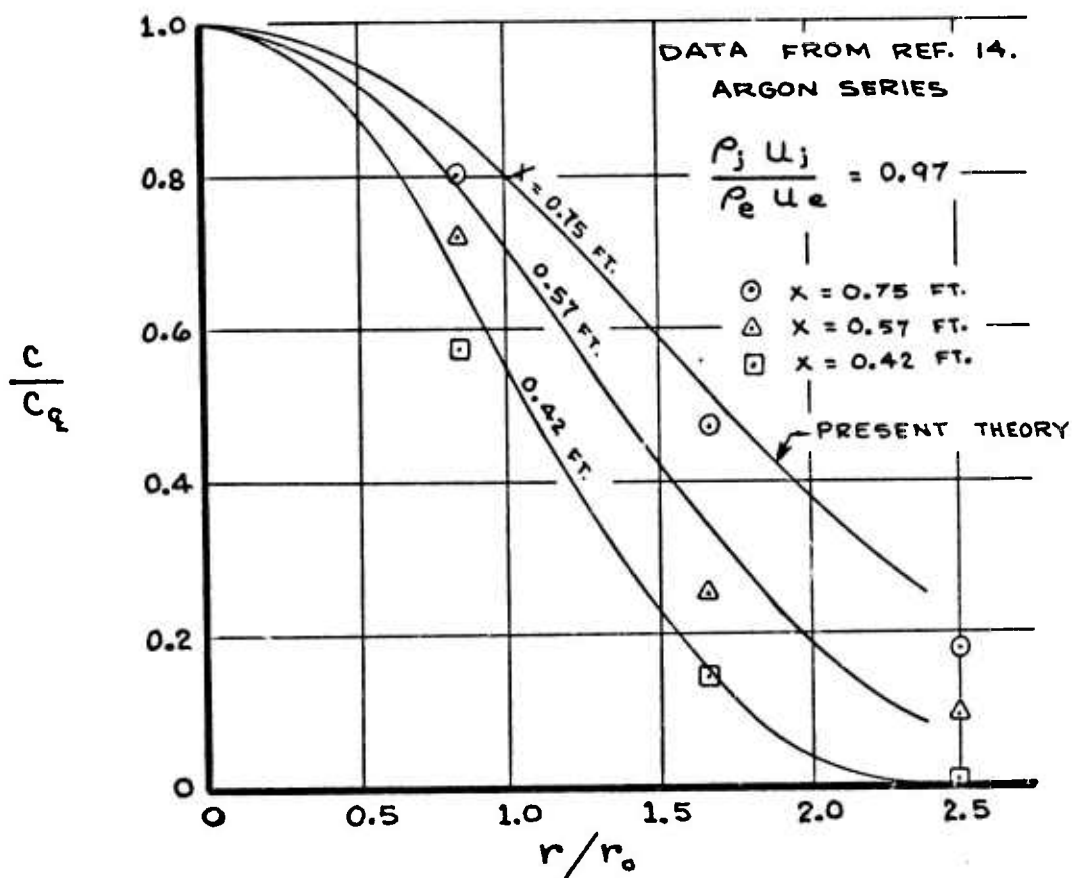
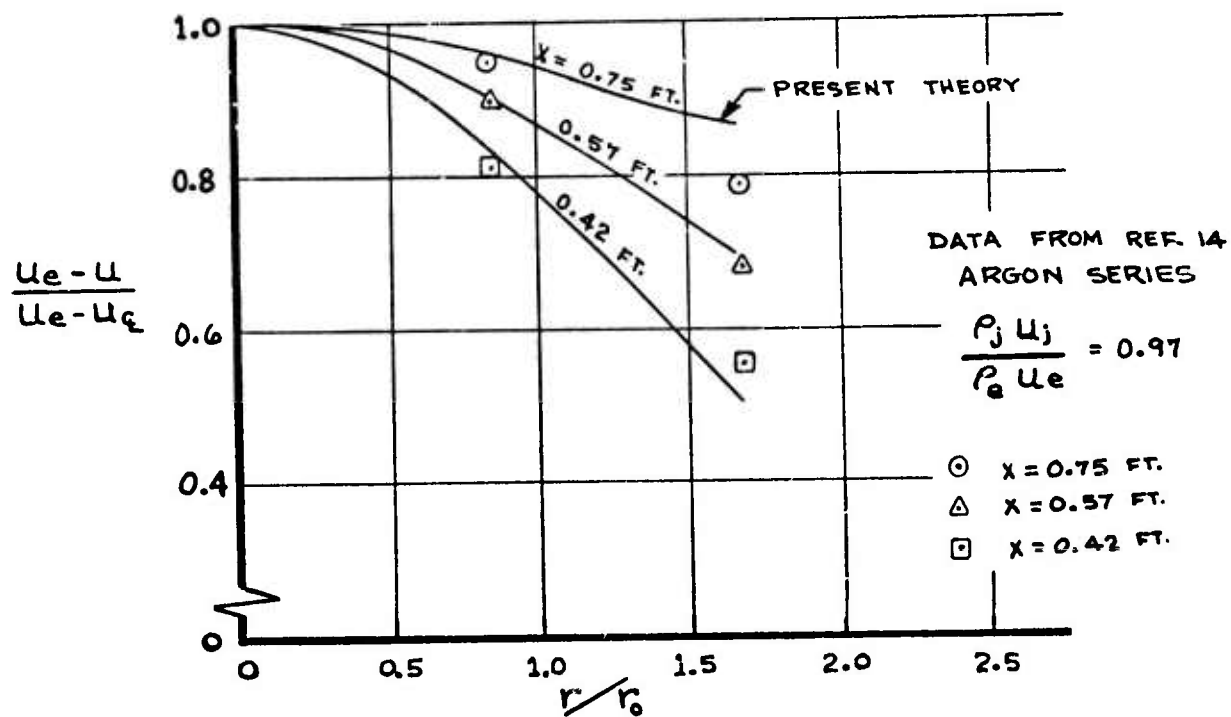


Fig. 14 COMPARISON OF THEORY AND EXPERIMENT

7.3 Sample Problem

In order to illustrate the utility of the computer program the solution of a typical air-augmentation problem is presented. Consider a rocket motor with the following operating conditions:

propellant	-	solid (16% Al)
nozzle exit radius	-	0.63 ft.
exit velocity	-	5570 ft/sec
exit static pressure	-	45,000.0 psfa
exit static temperature	-	4385°R
density	-	0.035 lbm/ft ³
Mach number	-	1.5

The secondary stream is assumed to be air with the following flow conditions at the exit plane of the nozzle:

static temperature	-	1094°R
static pressure	-	11992 psfa
velocity	-	856 ft/sec
duct diameter	-	2.52 ft.

It is desired to perform the calculations over an axial distance of 1.575 ft. The duct is assumed to be constant area. The preparation of the input data, presented in Fig. 15, is as follows.

NMIN

Since the flow properties of the jet are assumed constant at the nozzle exit plane, the value of NMIN is zero.

NMAX

NMAX is set equal to 196.

KSLIP

The value of KSLIP is taken to be

$$KSLIP = 129$$

NS

The value of NS is arbitrarily taken to be

$$NS = 3$$

NCHEM

It is desired to perform the calculations for equilibrium flow. Thus

$$NCHEM = 0$$

NCP8

Since it is not required to employ an additional species, other than those specified in the program, the value of NCP8 is

$$NCP8 = 0$$

NXTAB

The number of x-values in the XTAB table is

$$NXTAB = 6$$

INDUCT

It is desired to input the duct geometry, thus

$$INDUCT = 1$$

ISET

It is desired to continue the run beyond the point where the mixing zone crosses the reference streamline, thus

$$ISET = 1$$

DX

The value of DX is established from the stability criteria in the following manner. From Eqn. 94, 95 and 96 the initial value of ΔX required for stability is

$$\begin{aligned}\Delta X &\leq \frac{RE (U_{NMIN})^{1/2}}{2 KSLIP (2 \epsilon_0 Le/P_r)^{1/2}} \\ &= \frac{0.63 (5570)^{1/2}}{2 \cdot 129 \cdot (2 \cdot 4.95 \cdot 1.2/0.8)^{1/2}} \\ &= 0.0475 \text{ ft}\end{aligned}$$

The value of ΔX is taken somewhat smaller than the above value in order to maintain a sufficiently large number of grid points in the mixing region (see Section 5.5). Thus

$$DX = 0.005 \text{ ft.}$$

XMAX

The value of XMAX is

$$XMAX = 1.575 \text{ ft.}$$

XPRINT

The desired increment between printout of data is

USE FOR TYPEWRITTEN MATERIAL ONLY

$$XPRINT = 0.25 \text{ ft.}$$

RZERO

Since the static pressure of the secondary stream is considerably less than that of the rocket exhaust gases it is desired that the reference streamline be located along the plume slipline. Thus

$$RZERO = 0.001 \text{ ft.}$$

RE

The radius of the nozzle at the exit is

$$RE = 0.63 \text{ ft.}$$

RDZERO

The initial value of the duct radius is

$$RDZERO = 2.52 \text{ ft.}$$

PZERO

The initial pressure of the secondary stream is

$$PZERO = 11992.0 \text{ psfa}$$

DPDXO

The initial pressure gradient is obtained from the method of characteristics program employed to determine the slipline coordinates (5). Thus

$$DPDXO = -800.0 \text{ psfa/ft.}$$

DELTA

The flow is axisymmetric, thus

$$DELTA = 1.0$$

TESMAX and TESMIN

The values of TESMAX and TESMIN are taken to be

$$\text{TESMAX} = \text{TESMIN} = 0.00001$$

A, FL, EPSON, FCON

It is assumed that the eddy viscosity expression is of the form

$$\epsilon_v = \epsilon_o X$$

Thus, from Eqn. 94 it is seen that

$$A = 1.0$$

$$FL = 1.0$$

$$FCON = 0$$

The value of EPSON is determined from the equation

$$\text{EPSON} = \epsilon_o = \frac{\bar{U}}{2 \sigma^2}$$

where σ is determined from Figures 6 and 7, and \bar{U} is the average velocity between the two streams. Thus

$$\bar{U} = \frac{856 + 5570}{2}$$

$$= 3213 \text{ ft/sec}$$

From Figures 6 and 7 the values of σ_{II}/σ_I and σ_{II} are determined to be

$$\sigma_{II}/\sigma_I = 1.2$$

$$\sigma_{II} = 15 \times 1.2 = 18 \text{ ft}^{-\frac{1}{2}}$$

which yields

$$\epsilon_o = \frac{3213}{2 \times 18^2}$$

$$= 4.95 \text{ ft}^2/\text{sec}$$

PRT, FLETG, FLETP

These values are assumed to be

$$PRT = 0.8$$

$$FLETG = 1.2$$

$$FLETP = 0.6$$

XTAB, RTAB, PTAB

The values of XTAB and RTAB are determined from the method of characteristics program (5). The values of PTAB are the duct radii at the various XTAB's. For the present problem PTAB is assumed constant at

$$PTAB = 2.52 \text{ ft.}$$

GENN, W8, TCP8, CP8

Since a general species is not required the input for these parameters is omitted.

C_{i1} and C_{iNMAX}

The concentration of the species in the rocket exhaust are determined from an equilibrium thermochemical program (27). The secondary stream is assumed to be air.

T_1 and V

The temperatures and velocities are as follows:

$$T_1 = 4385^\circ R$$

$$T_{NMAX} = 1094^\circ R$$

$$U_1 = 5570 \text{ ft/sec}$$

$$U_{NMAX} = 856 \text{ ft/sec}$$

The input form for the above sample problem is presented in Fig. 15.

12A6		TITLE BC-3276-2									
1514		NMIN	NMAX	KSLIP	NS	NCHEN	NCP8	NXTAB	INDUCT	1SET	(Keypunch - right adjust)
8E10.0		DX	XMAX	XPRINT	RZERO1	RE	RDZERO	PZERO	DPDX0		
8E10.0		DELTA	TESMAX	TESMIN	A	FL	EPSOM	ECON	PRT		
2E10.0		FLETG	FLETP								
3E10.0		XTAB	RTAB	PTAB							
A6,4XE10.0		GENN <input checked="" type="checkbox"/> W8									
2E10.0		CPR <input checked="" type="checkbox"/> PR									
8E10.0		C1	C2	C3	C4	C5	C6	C7	C8	Input if and only if NCP8 > 0.	
4E10.0		Use a separate sheet to input the species concentration for the first eight species with N running from NMIN to NMAX.									
1E10.0		C9	C10	T	U	Input if and only if NMIN is input > 0.					
8E10.0		Use a separate sheet to input the last two species concentration and the temperature and the X-component of velocity for N running from NMIN to NMAX.									
8E10.0		XZERO									
8E10.0		C1	C2	C3	C4	C5	C6	C7	C8	Input if and only if NMIN is input < 0.	
8E10.0		C9	C10	C1 NMAX	C2 NMAX	C3 NMAX	C4 NMAX	C5 NMAX	C6 NMAX		
8E10.0		C7 NMAX	C8 NMAX	C9 NMAX	C10 NMAX	T	T NMAX	U	U NMAX		
8E10.0						4385.0	1094.0	5570.0	856.0		
8E10.0		CALC			REVISED	DATE	INPUT SHEET				
8E10.0		CHECK					Air Augmented Rocket Mixing Program				
8E10.0		APR					THE BOEING COMPANY				PAGE
8E10.0		APR									

U3 4038 8000

Figure 15 - Input Form for Sample Problem

REV LTR

U3 4288-2000 REV. 6/64

INITIAL INPUT

BC-3276-2

NMIN	NMAX	KSLIP	NS	NCHEV	NCPA	NXTAB	INDUCT	MPAY	LFRONT	ISEY
123	134	120	3	-n	0	6	1	501	50	1
DX	5000000-02	1.5750000		YPRINT	R7EP01	.1000000-02	.630000004	RE	RDZFR0	PZLFO
DELTA	1.0000000	.1000000-04		TESMIN	A	1.0000000	1.0000000	FL	EPSON	FCUN
FLETO	1.2000000	.60000004		ETA		1.0000000				PRT
										.500000004
XTAB	.0000000	.63000004		PTAB						
	.1000000-01	.64000005								
	.800000005	.67000003								
	1.5800000	1.2100000								
	2.5000000	1.3100000								
DP51	.30223922	.0000000		PS10						

PROPERTIES GIVEN BELOW ARE AT FRONT 1 DISTANCE DOWNSTREAM = .0000FEET PRESSURE= 11991.9999PSF

 INITIAL FLOW IN SECONDARY STREAM = 3275.4638 LBS/SEC THRUST = .0000 LB. FUCT AREA = 19.9504 50 FT.
 RADIUS OF DUCT = 2.5200 FT.

N	Y	X-COMP VELOCITY	TEMPERATURE	DENSITY	MACH NUMBER
1	.0000	5570.0	4384.9	.3496-01	1.4804
123	.61549	5570.0	4384.9	.3496-01	1.4804
124	.61801	5570.0	4384.9	.3496-01	1.4804
125	.62051	5570.0	4384.9	.3496-01	1.4804
126	.62301	5570.0	4384.9	.3496-01	1.4804
127	.62547	5220.8	4141.2	.3825-01	1.4549
128	.62783	4347.8	3531.7	.4650-01	1.3582
129	.63005	3213.0	2730.5	.6869-01	1.1909
130	.63221	2078.1	1947.2	.1054-00	.04304
131	.63450	1205.1	1337.7	.16334	.67377
132	.63708	856.00	1094.0	.20458	.53248
133	.63980	856.00	1094.0	.20458	.53248
134	.64240	856.70	1094.0	.20458	.53248
135	.64517	856.00	1094.0	.20458	.53248

N - - - - - CONCENTRATION OF SPECIES - - - - -

BOEING

NO.
SH.

D2-36251-1

94

REV LTR

U3 4288-2000 REV. 6/64

	H2O	H2	O2	CO2	CO	N2	HCL	AL	AL2O3
1	.9140-01	.2260-01	-.0000	.2390-01	.28204	.4510-01	.19304	-.0000	.30205
123	.9140-01	.2260-01	-.0000	.2390-01	.28204	.4510-01	.19304	-.0000	.30205
124	.9140-01	.2260-01	-.0000	.2390-01	.28204	.4510-01	.19304	-.0000	.30205
125	.9140-01	.2260-01	-.0000	.2390-01	.28204	.4510-01	.19304	-.0000	.30205
126	.9140-01	.2260-01	-.0000	.2390-01	.28204	.4510-01	.19304	-.0000	.30205
127	.8463-01	.2093-01	.1726-01	.2213-01	.26116	.13566	.17475	-.0000	.22375
128	.6770-01	.1674-01	.6041-01	.1770-01	.20893	.24193	.14301	-.0000	.15105
129	.4570-01	.1130-01	.1165-00	.1195-01	.14104	.42609	.5004-01	-.0000	.7830-01
130	.2370-01	.5359-02	.17264	.6198-02	.7311-01	.59026	.1430-01	-.0000	.2237-01
131	.6770-02	.1674-02	.21579	.1770-02	.2089-01	.71053	-.0000	-.0000	-.0000
132	-.0000	-.0000	.23304	-.0000	-.0000	.74704	-.0000	-.0000	-.0000
133	-.0000	-.0000	.23304	-.0000	-.0000	.74704	-.0000	-.0000	-.0000
134	-.0000	-.0000	.23304	-.0000	-.0000	.74704	-.0000	-.0000	-.0000
135	-.0000	-.0000	.23304	-.0000	-.0000	.74704	-.0000	-.0000	-.0000

DELTA PSI WAS SET = .60452

DELTA PSI WAS SET = 1.2090

PROPERTIES GIVEN BELOW ARE AT FRONT 51 DISTANCE DOWNSTREAM = .2500 FEET PRESSURE = 11730.5382 PSF
 RADIUS OF DUCT = 2.5202 FT. THRUST = 31.6754 LB. DUCT AREA = 19.951 SQ FT.

N	Y	X-COMP VELOCITY	TEMPERATURE	DENSITY	MACH NUMBER
1	.43375	5613.4	4362.5	.3437-01	1.5017
26	.70915	5613.4	4362.5	.3437-01	1.5017
27	.71695	5613.4	4362.5	.3437-01	1.5017
28	.72564	5613.4	4362.5	.3437-01	1.5016
29	.73422	5613.1	4362.8	.3437-01	1.4999
30	.74270	5607.3	4367.4	.3437-01	1.4903
31	.75117	5536.5	4414.5	.3435-01	1.3485
32	.75975	5024.9	4707.8	.3453-01	1.3485
33	.77083	3072.7	5235.5	.3900-01	.8578
34	.78476	1445.2	2898.6	.7645-01	.57441
35	.79730	1067.8	1733.8	.12735	.53993
36	.80760	960.84	1345.3	.16352	.54454
37	.81564	923.55	1161.5	.18420	.55293
38	.82515	909.90	1127.3	.19443	.55845
39	.83331	905.17	1101.4	.19680	.56130
40	.84130	903.71	1091.9	.20053	.56252
41	.84918	903.32	1082.8	.20107	.56297
42	.85690	903.23	1084.0	.20121	.56311
43	.86471	903.21	1087.8	.20121	.56314
44	.87237	903.21	1087.7	.20127	.56314
45	.87997	903.21	1087.7	.20127	.56314

N - - - - - CONCENTRATION OF SPECIES - - - - -

BOEING

NO.
SH.

D2-36251-1

95

FIG. 16 (CONT'D)

REV LTR

U3 4288-2000 REV. 6/64

BOEING

NO.

SH.

D2-36251-1

96

FIG. 16 (CONT'D)

DELTA PSI WAS SET = 2.4179													
PROPERTIES GIVEN BELOW ARE AT FRONT 101 DISTANCE DOWNSTREAM = .5000 FEET PRESSURE = 11444.9935 PSF													
RADIUS OF NOCT = 2.5200 FT. THRUST = 7.5641 LB. NOCT AREA = 19.9510 SQ FT.													
N	Y	X-COMP	VELOCITY	TEMPERATURE	DENSITY	WACH	NUMBER	CO	H2	HCL	AL	AL2O3	
1	.59348	5061.3	4337.5	3373-01	1.5187			.28204	.4510-01	.19304	.0000	.30205	
10	.76170	5061.3	4337.5	3373-01	1.5187			.28204	.4510-01	.19304	.0000	.30205	
11	.77815	5061.3	4337.5	3373-01	1.5187			.28204	.4510-01	.19304	.0000	.30205	
12	.79426	5061.3	4337.5	3373-01	1.5186			.28204	.4510-01	.19304	.0000	.30205	
13	.81008	5061.1	4337.8	3372-01	1.5169			.28204	.4510-01	.19304	.0000	.30204	
14	.82552	5050.2	4343.6	3361-01	1.4949			.28204	.4510-01	.19304	.0000	.30189	
15	.84087	5050.1	4404.9	3335-01	1.3512			.28204	.4510-01	.19304	.0000	.30189	
16	.85636	5061.7	4761.2	3750-01	.89508			.28204	.4510-01	.19304	.0000	.30189	
17	.87555	3125.5	5342.7	7433-01	.59311			.28204	.4510-01	.19304	.0000	.30189	
18	.90110	1404.0	2912.2	1222-00	.6301			.28204	.4510-01	.19304	.0000	.30189	
19	.92260	1125.0	1763.5	15030	.57109			.28204	.4510-01	.19304	.0000	.30189	
20	.94014	1018.3	1374.1	17083	.58173			.28204	.4510-01	.19304	.0000	.30189	
21	.95552	979.40	1211.6	18003	.58063			.28204	.4510-01	.19304	.0000	.30189	
22	.96967	902.71	1137.8	19359	.59257			.28204	.4510-01	.19304	.0000	.30189	
23	.98314	955.51	1104.2	19009	.59425			.28204	.4510-01	.19304	.0000	.30189	
24	.99530	954.09	1020.7	19009	.59425			.28204	.4510-01	.19304	.0000	.30189	
25	1.0091	953.27	1083.8	19710	.5957			.28204	.4510-01	.19304	.0000	.30189	
26	1.0214	953.02	1081.7	19747	.5957			.28204	.4510-01	.19304	.0000	.30189	
27	1.0343	952.95	1081.1	1958	.59588			.28204	.4510-01	.19304	.0000	.30189	

REV LTR

U3 4288-2000 REV. 6/64

CONCENTRATION OF SPECIES									
N	H2O	H2	O2	CO2	CO	HCL	AL	AL2O3	
2A	1.0456	952.94	1080.9	.19761	.59591				
2B	1.0588	952.94	1080.9	.19762	.59591				
3A	1.0700	952.94	1080.9	.19762	.59591				
1	.9140-01	.2260-01	.0000	.2390-01	.28204	.0000	.0000	.0000	
10	.9140-01	.2260-01	.0000	.2390-01	.28204	.0000	.0000	.0000	
11	.9140-01	.2260-01	.0000	.2390-01	.28204	.0000	.0000	.0000	
12	.9140-01	.2260-01	.0000	.2390-01	.28204	.0000	.0000	.0000	
13	.9140-01	.2260-01	.0000	.2390-01	.28204	.0000	.0000	.0000	
14	.9140-01	.2260-01	.0000	.2390-01	.28204	.0000	.0000	.0000	
15	.9140-01	.2260-01	.0000	.2390-01	.28204	.0000	.0000	.0000	
16	.9140-01	.2260-01	.0000	.2390-01	.28204	.0000	.0000	.0000	
17	.9140-01	.2260-01	.0000	.2390-01	.28204	.0000	.0000	.0000	
18	.9140-01	.2260-01	.0000	.2390-01	.28204	.0000	.0000	.0000	
19	.9140-01	.2260-01	.0000	.2390-01	.28204	.0000	.0000	.0000	
20	.9140-01	.2260-01	.0000	.2390-01	.28204	.0000	.0000	.0000	
21	.9140-01	.2260-01	.0000	.2390-01	.28204	.0000	.0000	.0000	
22	.9140-01	.2260-01	.0000	.2390-01	.28204	.0000	.0000	.0000	
23	.9140-01	.2260-01	.0000	.2390-01	.28204	.0000	.0000	.0000	
24	.9140-01	.2260-01	.0000	.2390-01	.28204	.0000	.0000	.0000	
25	.9140-01	.2260-01	.0000	.2390-01	.28204	.0000	.0000	.0000	
26	.9140-01	.2260-01	.0000	.2390-01	.28204	.0000	.0000	.0000	
27	.9140-01	.2260-01	.0000	.2390-01	.28204	.0000	.0000	.0000	
28	.9140-01	.2260-01	.0000	.2390-01	.28204	.0000	.0000	.0000	
29	.9140-01	.2260-01	.0000	.2390-01	.28204	.0000	.0000	.0000	
30	.9140-01	.2260-01	.0000	.2390-01	.28204	.0000	.0000	.0000	

11062.7586PSF

PRESSURE=

.750FEET

DISTANCE DOWNSTREAM =

151

ARE AT FRONT

PROPERTIES GIVEN BELOW ARE AT FRONT

RADIUS OF NOCT =

2.5198 FT.

THRUST =

-31.7370 LB.

NOCT AREA =

19.9475 SQ FT.

NO.

SH.

D2-36251-1

97

FIG. 16 (CONT'D)

REV LTR

U3 4288-2000 REV. 6/64

BOEING

NO.

D2-36251-1

SH.

98

CONCENTRATION OF SPECIES									
N	H2O	H2	O2	CO	N2	HCL	AL	AL2O3	
1	9140-01	2260-01	0000	28204	0510-01	19304	0000	0000	30205
2	9140-01	2260-01	0000	28204	0510-01	19304	0000	0000	30205
3	9140-01	2260-01	0000	28204	0510-01	19304	0000	0000	30205
4	9140-01	2260-01	0000	28204	0510-01	19304	0000	0000	30205
5	9140-01	2260-01	0000	28204	0510-01	19304	0000	0000	30205
6	9140-01	2260-01	0000	28204	0510-01	19304	0000	0000	30205
7	9140-01	2260-01	0000	28204	0510-01	19304	0000	0000	30205
8	9140-01	2260-01	0000	28204	0510-01	19304	0000	0000	30205
9	9140-01	2260-01	0000	28204	0510-01	19304	0000	0000	30205
10	9140-01	2260-01	0000	28204	0510-01	19304	0000	0000	30205
11	9140-01	2260-01	0000	28204	0510-01	19304	0000	0000	30205
12	9140-01	2260-01	0000	28204	0510-01	19304	0000	0000	30205
13	9140-01	2260-01	0000	28204	0510-01	19304	0000	0000	30205
14	9140-01	2260-01	0000	28204	0510-01	19304	0000	0000	30205
15	9140-01	2260-01	0000	28204	0510-01	19304	0000	0000	30205
16	9140-01	2260-01	0000	28204	0510-01	19304	0000	0000	30205
17	9140-01	2260-01	0000	28204	0510-01	19304	0000	0000	30205
18	9140-01	2260-01	0000	28204	0510-01	19304	0000	0000	30205
19	9140-01	2260-01	0000	28204	0510-01	19304	0000	0000	30205
20	9140-01	2260-01	0000	28204	0510-01	19304	0000	0000	30205
21	9140-01	2260-01	0000	28204	0510-01	19304	0000	0000	30205
22	9140-01	2260-01	0000	28204	0510-01	19304	0000	0000	30205
23	9140-01	2260-01	0000	28204	0510-01	19304	0000	0000	30205
24	9140-01	2260-01	0000	28204	0510-01	19304	0000	0000	30205
25	9140-01	2260-01	0000	28204	0510-01	19304	0000	0000	30205
26	9140-01	2260-01	0000	28204	0510-01	19304	0000	0000	30205
27	9140-01	2260-01	0000	28204	0510-01	19304	0000	0000	30205
28	9140-01	2260-01	0000	28204	0510-01	19304	0000	0000	30205
29	9140-01	2260-01	0000	28204	0510-01	19304	0000	0000	30205
30	9140-01	2260-01	0000	28204	0510-01	19304	0000	0000	30205
31	9140-01	2260-01	0000	28204	0510-01	19304	0000	0000	30205
32	9140-01	2260-01	0000	28204	0510-01	19304	0000	0000	30205
33	9140-01	2260-01	0000	28204	0510-01	19304	0000	0000	30205
34	9140-01	2260-01	0000	28204	0510-01	19304	0000	0000	30205
35	9140-01	2260-01	0000	28204	0510-01	19304	0000	0000	30205
36	9140-01	2260-01	0000	28204	0510-01	19304	0000	0000	30205
37	9140-01	2260-01	0000	28204	0510-01	19304	0000	0000	30205

REV LTR

U3 4288-2000 REV. 6/64

BOEING

NO.

SH.

D2-36251-1

99

PROPERTIES GIVEN BELOW ARE AT FRONT										701	DISTANCE DOWNSTREAM =		1.000 FEET	PRFSSURE =	10584.8160 PSF
RADIUS OF NOCT =										2.5200 FT.	THRUST =	-3.7467 LB.	DUCT AREA =	19.9500 SQ FT.	
N	Y	X-COMP VELOCITY	TEMPERATURE	DENSITY	MACH NUMBER										
1	.83190	5808.8	4259.0	.3177-01	1.5719										
5	.89277	5808.8	4259.0	.3177-01	1.5719										
6	.90733	5808.8	4259.0	.3177-01	1.5718										
7	.92166	5808.8	4259.1	.3177-01	1.5718										
8	.93577	5808.7	4259.2	.3176-01	1.5718										
9	.94967	5808.3	4260.0	.3176-01	1.5717										
10	.96338	5806.5	4262.9	.3176-01	1.5709										
11	.97690	5799.3	4273.0	.3173-01	1.5681										
12	.99030	5774.1	4302.1	.3165-01	1.5569										
13	1.0036	5700.2	4375.2	.3146-01	1.5338										
14	1.0172	5515.4	4535.7	.3113-01	1.4706										
15	1.0315	5115.6	4851.7	.3074-01	1.3639										
16	1.0473	4373.8	5244.8	.3146-01	1.1800										
17	1.0654	3502.2	5264.5	.3540-01	.9345										
18	1.0850	2513.4	4346.0	.4570-01	.75712										
19	1.1061	1759.0	3077.3	.6301-01	.67990										
20	1.1241	1486.3	2333.5	.8367-01	.65412										
21	1.1401	1341.0	1903.1	.1048+00	.64871										
22	1.1544	1256.6	1638.3	.1218+00	.63121										
23	1.1676	1204.0	1466.0	.13567	.62647										
24	1.1799	1169.8	1340.0	.14723	.62331										
25	1.1915	1146.4	1267.3	.15655	.61784										
26	1.2025	1130.4	1208.9	.16396	.61209										
27	1.2132	1113.2	1166.7	.16976	.60772										
28	1.2235	1111.3	1136.1	.17424	.60312										
29	1.2336	1105.8	1113.7	.17765	.60279										
30	1.2435	1102.0	1097.5	.18020	.60208										
31	1.2532	1099.3	1085.8	.18209	.60200										
32	1.2628	1097.5	1077.4	.18347	.60172										
33	1.2723	1096.2	1071.5	.18446	.60102										
34	1.2816	1095.4	1067.3	.18515	.60029										
35	1.2900	1094.9	1064.4	.18563	.60076										
36	1.3000	1094.5	1062.4	.18596	.60069										
37	1.3091	1094.3	1061.1	.18618	.60032										
38	1.3182	1094.1	1060.3	.18633	.60047										
39	1.3271	1094.1	1059.7	.18642	.60057										
40	1.3360	1094.0	1059.4	.18647	.60064										
41	1.3440	1094.0	1059.2	.18650	.60067										
42	1.3537	1094.0	1059.1	.18652	.60069										
43	1.3624	1094.0	1059.1	.18653	.60070										
44	1.3711	1094.0	1059.1	.18653	.60070										
45	1.3797	1094.0	1059.1	.18653	.60070										
CONCENTRATION OF SPECIES															
N	H2O	H2	O2	CU2	CO	HCL	AL	AL2O3							
1	.9140-01	.2260-01	-.0000	.2300-01	.28204	.19304	.0000	-.0000							
5	.9140-01	.2260-01	-.0000	.2300-01	.28204	.19304	.0000	-.0000							
6	.9140-01	.2260-01	-.0000	.2300-01	.28204	.19304	.0000	-.0000							

FIG. 16 (CONT'D)

RADIUS OF NUCT =	2.5201 FT.	THRUST =	14.5053 LB.	NUCT AREA =	19.9517 SQ FT.
------------------	------------	----------	-------------	-------------	----------------

REV LTR

U3 4288-2000 REV. 6/64

N	Y	X-COMP VELOCITY	TEMPERATURE	DENSITY	WACH NUMBER
1	.86405	5041.6	4241.2	.3134-01	1.5841
2	.89410	5041.6	4241.2	.3133-01	1.5839
3	.92317	5041.6	4241.2	.3133-01	1.5839
4	.95136	5041.5	4241.4	.3133-01	1.5833
5	.97975	5040.3	4243.7	.3133-01	1.5769
6	1.0054	5043.7	4266.0	.3127-01	1.5316
7	1.0318	5091.0	4390.1	.3092-01	1.3514
8	1.0595	5059.7	4902.4	.3027-01	1.3514
9	1.0935	3341.5	5251.9	.3447-01	.96707
10	1.1322	1869.5	3240.6	.6064-01	.70523
11	1.1652	1419.8	2010.5	.9712-01	.66925
12	1.1936	1262.8	1541.1	.12083	.67303
13	1.2174	1104.0	1316.2	.14018	.68377
14	1.2369	1159.9	1194.1	.16249	.69311
15	1.2590	1142.1	1132.8	.17165	.69990
16	1.2762	1132.7	1096.2	.17725	.70445
17	1.2968	1127.8	1075.8	.18051	.70731
18	1.3140	1125.4	1064.8	.18232	.70900
19	1.3327	1124.2	1054.0	.18328	.70995
20	1.3502	1123.7	1056.2	.18375	.71044
21	1.3674	1123.5	1054.8	.18397	.71068
22	1.3844	1123.4	1054.3	.18406	.71079
23	1.4012	1123.4	1054.1	.18409	.71082
24	1.4178	1123.4	1054.0	.18410	.71083
25	1.4342	1123.4	1054.0	.18410	.71083

N	H2O	H2	C2	CO	N2	HCL	AL	AL2O3
1	.9140-01	.2260-01	.0000	.28204	.4510-01	.19304	.0000	.30205
2	.9140-01	.2260-01	.0000	.28204	.4510-01	.19304	.0000	.30205
3	.9140-01	.2260-01	.0000	.28204	.4510-01	.19304	.0000	.30205
4	.9140-01	.2260-01	.0000	.28204	.4510-01	.19304	.0000	.30205
5	.9140-01	.2260-01	.0000	.28204	.4510-01	.19304	.0000	.30205
6	.9140-01	.2260-01	.0000	.28204	.4510-01	.19304	.0000	.30205
7	.9140-01	.2260-01	.0000	.28204	.4510-01	.19304	.0000	.30205
8	.9140-01	.2260-01	.0000	.28204	.4510-01	.19304	.0000	.30205
9	.9140-01	.2260-01	.0000	.28204	.4510-01	.19304	.0000	.30205
10	.9140-01	.2260-01	.0000	.28204	.4510-01	.19304	.0000	.30205
11	.9140-01	.2260-01	.0000	.28204	.4510-01	.19304	.0000	.30205
12	.9140-01	.2260-01	.0000	.28204	.4510-01	.19304	.0000	.30205
13	.9140-01	.2260-01	.0000	.28204	.4510-01	.19304	.0000	.30205
14	.9140-01	.2260-01	.0000	.28204	.4510-01	.19304	.0000	.30205
15	.9140-01	.2260-01	.0000	.28204	.4510-01	.19304	.0000	.30205
16	.9140-01	.2260-01	.0000	.28204	.4510-01	.19304	.0000	.30205
17	.9140-01	.2260-01	.0000	.28204	.4510-01	.19304	.0000	.30205
18	.9140-01	.2260-01	.0000	.28204	.4510-01	.19304	.0000	.30205
19	.9140-01	.2260-01	.0000	.28204	.4510-01	.19304	.0000	.30205
20	.9140-01	.2260-01	.0000	.28204	.4510-01	.19304	.0000	.30205
21	.9140-01	.2260-01	.0000	.28204	.4510-01	.19304	.0000	.30205
22	.9140-01	.2260-01	.0000	.28204	.4510-01	.19304	.0000	.30205
23	.9140-01	.2260-01	.0000	.28204	.4510-01	.19304	.0000	.30205
24	.9140-01	.2260-01	.0000	.28204	.4510-01	.19304	.0000	.30205
25	.9140-01	.2260-01	.0000	.28204	.4510-01	.19304	.0000	.30205

BOEING

NO.

D2-36251-1

SH.

101

REV LTR

U3 4288-2000 REV. 6/64

BOEING

NO.

D2-36251-1

SH.

103

TRANSFORMATION COMPLETED

PROPERTIES GIVEN BELOW ARE AT FRONT 216 DISTANCE DOWNSTREAM = 1.0750 FEET PRESSURE = 10397.4958 PSF
 RADIUS OF NOSE = 2.5201 FT. THROAT = 14.5853 LB. NOSE AREA = 19.9517 SQ FT.

N	Y	X-COMP VELOCITY	TEMPERATURE	DENSITY	MACH NUMBER
1	.6300-04	5841.6	4241.2	.3133-01	1.5839
13	.79816	5841.6	4241.2	.3133-01	1.5839
14	.83074	5841.6	4241.2	.3133-01	1.5839
15	.86211	5841.6	4241.2	.3133-01	1.5839
16	.89237	5841.6	4241.2	.3133-01	1.5839
17	.92161	5841.6	4241.2	.3133-01	1.5839
18	.95000	5841.5	4241.4	.3133-01	1.5839
19	.97754	5840.3	4243.7	.3133-01	1.5833
20	1.0043	5823.7	4266.0	.3127-01	1.5769
21	1.0308	5691.0	4399.1	.3092-01	1.5316
22	1.0584	5059.7	4902.4	.3027-01	1.3514
23	1.0928	3341.5	5251.9	.3497-01	.8677
24	1.1318	1663.5	3240.6	.6044-01	.70523
25	1.1680	1419.8	2019.5	.9712-01	.6825
26	1.1935	1262.8	1541.1	.1268	.67303
27	1.2174	1194.0	1316.2	.14818	.68377
28	1.2390	1153.9	1198.1	.16249	.69311
29	1.2592	1143.1	1132.8	.17165	.69930
30	1.2785	1132.7	1096.2	.17725	.70445
31	1.2972	1127.6	1075.8	.18051	.70731
32	1.3154	1123.4	1054.8	.18232	.70900
33	1.3332	1124.2	1059.0	.18328	.70935
34	1.3508	1123.7	1056.2	.18375	.71044
35	1.3681	1123.5	1054.8	.18397	.71068
36	1.3851	1123.4	1054.3	.18406	.71079
37	1.4020	1123.4	1054.1	.18409	.71082
38	1.4187	1123.4	1054.0	.18410	.71083
39	1.4351	1123.4	1054.0	.18410	.71083
40	1.4514	1123.4	1054.0	.18410	.71083
41	1.4675	1123.4	1054.0	.18410	.71083

N	H2O	H2	O2	CO2	CO	HCL	AL	AL2O3
1	.9140-01	.2260-01	.0000	.2390-01	.28204	.19304	.0000	.30205
13	.9140-01	.2260-01	.0000	.2390-01	.28204	.19304	.0000	.30205
14	.9140-01	.2260-01	.0000	.2390-01	.28204	.19304	.0000	.30205
15	.9140-01	.2260-01	.0000	.2390-01	.28204	.19304	.0000	.30205
16	.9140-01	.2260-01	.0000	.2390-01	.28204	.19304	.0000	.30205
17	.9140-01	.2260-01	.0000	.2390-01	.28204	.19304	.0000	.30205
18	.9140-01	.2260-01	.0000	.2390-01	.28204	.19304	.0000	.30205
19	.9164-01	.2252-01	.0000	.2415-01	.28135	.19270	.0000	.30203

FIG. 16 (CONT'D)

REV LTR

U3 4288-2000 REV. 6/64

PROPERTIES GIVEN BELOW ARE AT FRONT 266 DISTANCE DOWNSTREAM = 1.3250 FEET PRESSURE = 9686.0389 PSF
 RADIUS OF NOCT = 2.5201 FT. THRUST = 10.8137 LB. NOCT AREA = 19.9515 SQ FT.

N	Y	X-COMP	VELOCITY	TEMPERATURE	DENSITY	WACH	NUMBER
1	.34421	5968.7	4171.1	.2968-01	1.6313		
14	.91100	5968.7	4171.1	.2968-01	1.6313		
15	.94140	5968.7	4171.1	.2968-01	1.6313		
16	.97018	5968.7	4171.2	.2968-01	1.6312		
17	.99805	5968.6	4171.6	.2968-01	1.6312		
18	1.0251	5967.0	4174.4	.2967-01	1.6305		
19	1.05116	5955.9	4190.4	.2963-01	1.6038		
20	1.0776	5895.4	4257.5	.2945-01	1.5288		
21	1.1039	5852.8	4462.2	.2903-01	1.3457		
22	1.1317	4935.6	4802.4	.2930-01	1.0086		
23	1.1651	3485.2	5251.9	.3250-01	1.0086		
24	1.2025	2220.1	3670.4	.4962-01	.78977		
25	1.2360	1703.0	2391.0	.7044-01	.74209		
26	1.2634	1490.3	1803.5	.1012-00	.73909		
27	1.2875	1382.6	1494.4	.1215-00	.74655		
28	1.3098	1323.0	1324.2	.13726	.75575		
29	1.3296	1287.8	1218.3	.14097	.76397		
30	1.3471	1260.2	1151.4	.15744	.77056		
31	1.3644	1252.8	1104.4	.16342	.77554		
32	1.3820	1244.4	1090.6	.16752	.77918		
33	1.3986	1233.2	1062.7	.17026	.78174		

REV LTR

U3 4288-2000 REV. 6/64

BOEING

NO.
SH.

D2-36251-1

105

CONCENTRATION OF SPECIES										PRESSURE		8461.7002PSF	
N	H2O	H2	O2	CO2	CO	N2	HCL	AL	AL2O3				
34	1.4149	1236.1	1051.3	.17205	.78350	.4510-01	.19304	.0000	.30205				
35	1.4309	1234.2	1044.2	.17318	.78406	.4510-01	.19304	.0000	.30205				
36	1.4467	1233.1	1039.9	.17388	.78541	.4510-01	.19304	.0000	.30205				
37	1.4623	1232.5	1037.3	.17430	.78588	.4510-01	.19304	.0000	.30205				
38	1.4777	1232.2	1035.8	.17453	.78615	.4510-01	.19304	.0000	.30204				
39	1.4929	1232.0	1035.0	.17466	.78631	.4510-01	.19304	.0000	.30202				
40	1.5079	1232.0	1034.6	.17473	.78639	.4510-01	.19304	.0000	.30174				
41	1.5229	1231.9	1034.4	.17476	.78643	.4510-01	.19304	.0000	.30174				
42	1.5376	1231.9	1034.4	.17477	.78644	.4510-01	.19304	.0000	.30174				
43	1.5522	1231.9	1034.4	.17477	.78644	.4510-01	.19304	.0000	.30174				
44	1.5668	1231.9	1034.4	.17477	.78644	.4510-01	.19304	.0000	.30174				
45	1.5814	1231.9	1034.4	.17477	.78644	.4510-01	.19304	.0000	.30174				
46	1.5960	1231.9	1034.4	.17477	.78644	.4510-01	.19304	.0000	.30174				
47	1.6106	1231.9	1034.4	.17477	.78644	.4510-01	.19304	.0000	.30174				
48	1.6252	1231.9	1034.4	.17477	.78644	.4510-01	.19304	.0000	.30174				
49	1.6398	1231.9	1034.4	.17477	.78644	.4510-01	.19304	.0000	.30174				
50	1.6544	1231.9	1034.4	.17477	.78644	.4510-01	.19304	.0000	.30174				
51	1.6690	1231.9	1034.4	.17477	.78644	.4510-01	.19304	.0000	.30174				
52	1.6836	1231.9	1034.4	.17477	.78644	.4510-01	.19304	.0000	.30174				
53	1.6982	1231.9	1034.4	.17477	.78644	.4510-01	.19304	.0000	.30174				
54	1.7128	1231.9	1034.4	.17477	.78644	.4510-01	.19304	.0000	.30174				
55	1.7274	1231.9	1034.4	.17477	.78644	.4510-01	.19304	.0000	.30174				
56	1.7420	1231.9	1034.4	.17477	.78644	.4510-01	.19304	.0000	.30174				
57	1.7566	1231.9	1034.4	.17477	.78644	.4510-01	.19304	.0000	.30174				
58	1.7712	1231.9	1034.4	.17477	.78644	.4510-01	.19304	.0000	.30174				
59	1.7858	1231.9	1034.4	.17477	.78644	.4510-01	.19304	.0000	.30174				
60	1.8004	1231.9	1034.4	.17477	.78644	.4510-01	.19304	.0000	.30174				
61	1.8150	1231.9	1034.4	.17477	.78644	.4510-01	.19304	.0000	.30174				
62	1.8296	1231.9	1034.4	.17477	.78644	.4510-01	.19304	.0000	.30174				
63	1.8442	1231.9	1034.4	.17477	.78644	.4510-01	.19304	.0000	.30174				
64	1.8588	1231.9	1034.4	.17477	.78644	.4510-01	.19304	.0000	.30174				
65	1.8734	1231.9	1034.4	.17477	.78644	.4510-01	.19304	.0000	.30174				
66	1.8880	1231.9	1034.4	.17477	.78644	.4510-01	.19304	.0000	.30174				
67	1.9026	1231.9	1034.4	.17477	.78644	.4510-01	.19304	.0000	.30174				
68	1.9172	1231.9	1034.4	.17477	.78644	.4510-01	.19304	.0000	.30174				
69	1.9318	1231.9	1034.4	.17477	.78644	.4510-01	.19304	.0000	.30174				
70	1.9464	1231.9	1034.4	.17477	.78644	.4510-01	.19304	.0000	.30174				
71	1.9610	1231.9	1034.4	.17477	.78644	.4510-01	.19304	.0000	.30174				
72	1.9756	1231.9	1034.4	.17477	.78644	.4510-01	.19304	.0000	.30174				
73	1.9902	1231.9	1034.4	.17477	.78644	.4510-01	.19304	.0000	.30174				
74	2.0048	1231.9	1034.4	.17477	.78644	.4510-01	.19304	.0000	.30174				
75	2.0194	1231.9	1034.4	.17477	.78644	.4510-01	.19304	.0000	.30174				
76	2.0340	1231.9	1034.4	.17477	.78644	.4510-01	.19304	.0000	.30174				
77	2.0486	1231.9	1034.4	.17477	.78644	.4510-01	.19304	.0000	.30174				
78	2.0632	1231.9	1034.4	.17477	.78644	.4510-01	.19304	.0000	.30174				
79	2.0778	1231.9	1034.4	.17477	.78644	.4510-01	.19304	.0000	.30174				
80	2.0924	1231.9	1034.4	.17477	.78644	.4510-01	.19304	.0000	.30174				
81	2.1070	1231.9	1034.4	.17477	.78644	.4510-01	.19304	.0000	.30174				
82	2.1216	1231.9	1034.4	.17477	.78644	.4510-01	.19304	.0000	.30174				
83	2.1362	1231.9	1034.4	.17477	.78644	.4510-01	.19304	.0000	.30174				
84	2.1508	1231.9	1034.4	.17477	.78644	.4510-01	.19304	.0000	.30174				
85	2.1654	1231.9	1034.4	.17477	.78644	.4510-01	.19304	.0000	.30174				
86	2.1800	1231.9	1034.4	.17477	.78644	.4510-01	.19304	.0000	.30174				
87	2.1946	1231.9	1034.4	.17477	.78644	.4510-01	.19304	.0000	.30174				
88	2.2092	1231.9	1034.4	.17477	.78644	.4510-01	.19304	.0000	.30174				
89	2.2238	1231.9	1034.4	.17477	.78644	.4510-01	.19304	.0000	.30174				
90	2.2384	1231.9	1034.4	.17477	.78644	.4510-01	.19304	.0000	.30174				
91	2.2530	1231.9	1034.4	.17477	.78644	.4510-01	.19304	.0000	.30174				
92	2.2676	1231.9	1034.4	.17477	.78644	.4510-01	.19304	.0000	.30174				
93	2.2822	1231.9	1034.4	.17477	.78644	.4510-01	.19304	.0000	.30174				
94	2.2968	1231.9	1034.4	.17477	.78644	.4510-01	.19304	.0000	.30174				
95	2.3114	1231.9	1034.4	.17477	.78644	.4510-01	.19304	.0000	.30174				
96	2.3260	1231.9	1034.4	.17477	.78644	.4510-01	.19304	.0000	.30174				
97	2.3406	1231.9	1034.4	.17477	.78644	.4510-01	.19304	.0000	.30174				
98	2.3552	1231.9	1034.4	.17477	.78644	.4510-01	.19304	.0000	.30174				
99	2.3698	1231.9	1034.4	.17477	.78644	.4510-01	.19304	.0000	.30174				
100	2.3844	1231.9	1034.4	.17477	.78644	.4510-01	.19304	.0000	.30174				

PROPERTIES GIVEN BELOW ARE AT FRONT 116 DISTANCE DOWNSTREAM = 1.5750FEET PRESSURE = 8461.7002PSF

RADIUS OF NOCT = 2.519A FT. THRUST = -23.2979 LB. NOCT AREA = 19.9476 SQ FT.

REV LTR

U3 4288-2000 REV. 6/64

N	Y	X-COMP VELOCITY	TEMPERATURE	DENSITY	"ACH NUMBER
1	.41590	4040.1	.2677-01	1.7200	
13	.93586	6193.3	4040.1	1.7200	
14	.98544	6193.3	4040.1	1.7200	
15	.99624	6193.3	4040.2	1.7200	
16	1.0252	6193.1	4040.6	1.7199	
17	1.0534	6193.8	4043.2	1.7193	
18	1.0800	6199.7	4054.3	1.7191	
19	1.1074	6157.9	4092.9	1.7036	
20	1.1345	6044.9	4200.7	1.6649	
21	1.1614	5725.1	4434.4	1.5636	
22	1.1911	5005.5	4937.8	1.3820	
23	1.2244	3793.0	5141.4	1.1111	
24	1.2610	2691.4	3908.8	.92317	
25	1.2936	2111.7	2658.6	.8741	
26	1.3200	1634.8	2015.6	.80435	
27	1.3444	1093.0	1657.3	.7918-01	.86800
28	1.3658	1592.1	1441.0	.1103400	.87500
29	1.3853	1534.1	1302.1	.88344	
30	1.4035	1493.5	1200.2	.89002	
31	1.4207	1459.1	1145.3	.89608	
32	1.4372	1430.3	1100.6	.90101	
33	1.4531	1433.2	1060.0	.90551	
34	1.4686	1429.4	1046.7	.90833	
35	1.4838	1423.3	1031.0	.91042	
36	1.4997	1419.1	1016.9	.91253	
37	1.5133	1416.3	1012.2	.91379	
38	1.5277	1414.3	1007.0	.91409	
39	1.5420	1413.2	1003.4	.91532	
40	1.5561	1412.4	1001.1	.91576	
41	1.5700	1411.9	999.57	.91606	
42	1.5838	1411.6	998.60	.91625	
43	1.5975	1411.5	998.00	.91637	
44	1.6111	1411.4	997.66	.91644	
45	1.6245	1411.3	997.48	.91647	
46	1.6378	1411.3	997.40	.91649	
47	1.6511	1411.3	997.40	.91649	

N	H2O	H2	O2	CO2	CO	HCL	AL	AL2O3
1	.9140-01	.2260-01	.0000	.2390-01	.28204	.0000	.0000	.30205
13	.9140-01	.2260-01	.0000	.2390-01	.28204	.0000	.0000	.30205
14	.9140-01	.2260-01	.0000	.2390-01	.28204	.0000	.0000	.30205
15	.9140-01	.2260-01	.0000	.2390-01	.28204	.0000	.0000	.30205
16	.9144-01	.2259-01	.0000	.2394-01	.28194	.0000	.0000	.30204
17	.9157-01	.2250-01	.0000	.2422-01	.28123	.0000	.0000	.30202
18	.9251-01	.2220-01	.0000	.2519-01	.27872	.0000	.0000	.30185
19	.9502-01	.2130-01	.1472-05	.2602-01	.27130	.0000	.0000	.30080
20	.1014+00	.1976-01	.3638-05	.3476-01	.25321	.0000	.0000	.29586
21	.1144+00	.1443-01	.6368-05	.4843-01	.21606	.0000	.0000	.27797
22	.13160	.7079-02	.3750-03	.6910-01	.15303	.0000	.2439-06	.28861
23	.1155+00	.1044-02	.2896-01	.1235+00	.4433-01	.0000	.1005-06	.13453
24	.6540-01	.1134-04	.1065-00	.1040+00	.5102-03	.0000	.0000	.5565-01
25	.3744-01	.4411-05	.16145	.5907-01	.1997-03	.0000	.0000	.2402-01
26	.2341-01	.2177-05	.18416	.3007-01	.7-285	.0000	.0000	.1164-01
27	.1507-01	.1174-05	.20293	.2476-01	.5534-04	.0000	.0000	.6178-02

BOEING

NO. D2-36251-1

SH. 106

REV LTR

U3 4288-2000 REV. 6/64

BOEING

NO. D2-36251-1

SH.

107

28	.1094-01	.7310-05	.21199	.1729-01	.3578-04	.74171	.7169-02	.0000	.0000	.3440-02
29	.7830-02	.4704-06	.21794	.1239-01	.2491-04	.74090	.5136-02	.0000	.0000	.1973-02
30	.5697-02	.5204-06	.22206	.9004-02	.1907-04	.75386	.3732-02	.0000	.0000	.1149-02
31	.4170-02	.2090-06	.22500	.6589-02	.1359-04	.75739	.2732-02	.0000	.0000	.6723-03
32	.5057-02	.1250-06	.22714	.4831-02	.9997-05	.75997	.2003-02	.0000	.0000	.5923-03
33	.2234-02	.5914-07	.22872	.3533-02	.7452-05	.74187	.1405-02	.0000	.0000	.2270-03
34	.1627-02	.0000	.22930	.2571-02	.5515-05	.74323	.1766-02	.0000	.0000	.1297-03
35	.1174-02	.0000	.23077	.1350-02	.4035-05	.74433	.7694-03	.0000	.0000	.7294-04
36	.8392-03	.0000	.23142	.1320-02	.2911-05	.74510	.5498-03	.0000	.0000	.4028-04
37	.5921-03	.0000	.23190	.9355-03	.2056-05	.74567	.3479-03	.0000	.0000	.2180-04
38	.4115-03	.0000	.23225	.6501-03	.1438-05	.74609	.3596-03	.0000	.0000	.1155-04
39	.2804-03	.0000	.23250	.4433-03	.9797-06	.74643	.1334-03	.0000	.0000	.5970-05
40	.1857-03	.0000	.23264	.2949-03	.6436-06	.74681	.1223-03	.0000	.0000	.3005-05
41	.1200-03	.0000	.23281	.1373-03	.4137-05	.74677	.7863-04	.0000	.0000	.1463-05
42	.7330-04	.0000	.23290	.1153-03	.2527-06	.74688	.4005-04	.0000	.0000	.6793-06
43	.4094-04	.0000	.23297	.6471-04	.1404-06	.74695	.2683-04	.0000	.0000	.2904-06
44	.1805-04	.0000	.23301	.2051-04	.6107-07	.74700	.1182-04	.0000	.0000	.1016-06
45	.0000	.0000	.23304	.0000	.0000	.74704	.0000	.0000	.0000	.0000
46	.0000	.0000	.23304	.0000	.0000	.74704	.0000	.0000	.0000	.0000
47	.0000	.0000	.23304	.0000	.0000	.74704	.0000	.0000	.0000	.0000

DOCUMENT CONTROL DATA - R&D		
(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)		
1 ORIGINATING ACTIVITY (Corporate author) The Boeing Company Box 3707 Seattle, Washington 98124		2a REPORT SECURITY CLASSIFICATION Unclassified
		2b GROUP
3 REPORT TITLE ANALYSIS OF THE CHEMICALLY-REACTIVE MIXING REGION BETWEEN A TURBULENT ROCKET EXHAUST AND A CONFINED AIR STREAM		
4 DESCRIPTIVE NOTES (type of report and inclusive dates)		
5 AUTHOR(S) (Last name, first name, initial) Emmons, Donald L.		
6 REPORT DATE February 1965	7a TOTAL NO. OF PAGES 109	7b NO. OF REFS 28
8a CONTRACT OR GRANT NO.	9a ORIGINATOR'S REPORT NUMBER(S) D2-36251-1	
b PROJECT NO.		
c	9b OTHER REPORT NO(S) (Any other numbers that may be assigned this report)	
d		
10 AVAILABILITY/LIMITATION NOTICES Distribution of this report is Boeing-controlled. Qualif DDC users shall request through The Boeing Company, Aero-Space Division, Box 3707, Seattle, Washington 98124, Attention: Mr. [redacted]		
11 SUPPLEMENTARY NOTES		12 SPONSORING MILITARY ACTIVITY
13 ABSTRACT A theoretical analysis was developed for predicting the flow properties in the mixing region between a particle-laden, turbulent rocket exhaust and a surrounding air stream. It was assumed that the turbulent boundary layer equations, modified to account for particles were valid within the mixing region. In treating the chemical aspects of the problem it was assumed that the flow was in equilibrium. The chemical species comprising the flow were limited to the following: H ₂ O, H ₂ , O ₂ , CO, CO ₂ , N ₂ , HCl, Al, Al ₂ O ₃ , and one additional inert species. The phenomenological model employed for the turbulent transport coefficients is discussed in detail. The solution of the partial differential equations was obtained using the von Mises transformation, expressing the equations in finite-difference form, and solving the resulting equations utilizing a computer program developed for the SRU 1107. Results from the computer program were successfully compared with experimental results obtained from air-augmentation, free jet, and fuel injection experiments.		

DD FORM 1 JAN 64 1473
U3 4802 1030 ORIG. 12/64
PART 1 OF 2

REV LTR _____

Unclassified
Security Classification

BOEING NO. D2-36251-1
SH 108

Unclassified

Security Classification

14.	KEY WORDS	LINK A		LINK B		LINK C	
		ROLE	WT	ROLE	WT	ROLE	WT
	Gas-Particle Flow Mixing Analysis Chemical Reactions Turbulent Flow Eddy Transport Coefficients Air Augmented Rocket Supersonic Combustion						

DD FORM 1 NOV 65 1473
 U3 4802 1030 REV 7/66
 PART 2 OF 3

Unclassified

Security Classification

No. D2-36251-1

Sheet 109